The Formulas of Heron and Brahmagupta

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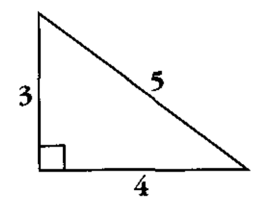
> *As the sun eclipses the stars by his brilliancy, so the man of knowledge will eclipse the fame of others in assemblies of the people if he proposes algebraic problems, and still more if he solves them.*
>
> —**Brahmagupta**

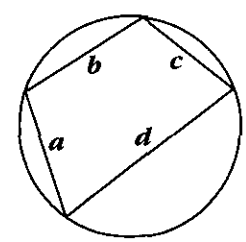
Egyptian mathematician **Heron of Alexandria** (ca A.D. 50–100) made considerable contributions to the development of applied mathematics. He is best known for his derivation of a famous formula for the area of a triangle in terms of its three sides: If a, b, and c are the sides of a triangle, and if $s = {}^{(a+b+c)}/_{2}$, then the area of the triangle is $\sqrt{s(s-a)(s-b)(s-c)}$ square units.*

While applying a formula to a sample problem doesn't prove the formula, we can check out whether Heron's formula works with a right triangle. The area of the right triangle shown at right is one half the product of its legs. That is, the area is $\frac{1}{2}(3)(4) = 6$ square units. Using Heron's formula, we see that $s = {}^{(3+4+5)}/_{2} = 6$, and the area of the triangle is $\sqrt{6(6-3)(6-4)(6-5)} = \sqrt{36} = 6$ square units.



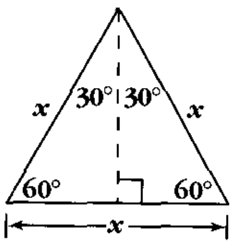
Heron's formula sometimes yields irrational values. For example, if you applied it to a triangle whose sides are 5, 6, and 7 units, then you would find that the triangle's area is $\sqrt{216}$ square units, an irrational result. Heron didn't share the belief of his Greek colleagues that irrational numbers could not exist, and that, in general, the product of more than three numbers was meaningless. In this sense, he was ahead of his time.

About 500 years later, Hindu mathematician **Brahmagupta** (b A.D. 598) discovered a formula similar to Heron's for the area of a cyclic quadrilateral: $\sqrt{(s-a)(s-b)(s-c)(s-d)}$, where $s = {}^{(a+b+c+d)}/_{2}$. (A cyclic quadrilateral is a quadrilateral that can be inscribed within a circle, as shown at right.) Brahmagupta's most important work was *Brahmasphutasiddhanta (Correct astronomical system of Brahma,* A.D. 628). In medieval India, most mathematical works were written as chapters of astronomy books, and the mathematical concepts and techniques were applied to astronomical problems. This was true of the *Brahmasphutasiddhanta*—and it was written completely in verse. (For more on the mathematics of India see vignettes 2, 24, 28, and 82.) ★

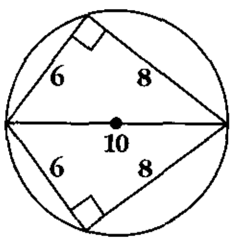


*See the proof for this theorem in the Bernard Oliver article listed in Related Reading.

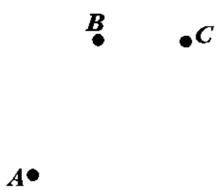
Activities



1. Use the properties of basic geometry to derive the formula for the area of an equilateral triangle in terms of a side x. Then use Heron's formula to determine the area in terms of x. Compare the area formulas you obtained by using these different methods.



2. The figure at left shows a cyclic quadrilateral inscribed within a circle with a diameter of 10. Calculate the area of the quadrilateral by adding the areas of the two triangles shown and applying Brahmagupta's formula.



3. Given the three points A, B, and C shown at left, find the collection of all points, D, such that the four points A, B, C, and D could be joined in some order to form a cyclic quadrilateral.

4. The area of a right triangle with legs of length a and b is $^{ab}/_2$. Use Heron's formula to derive this formula.

Related Reading

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