Demo 24: How the Width of the Sampling Distribution Depends on *N*

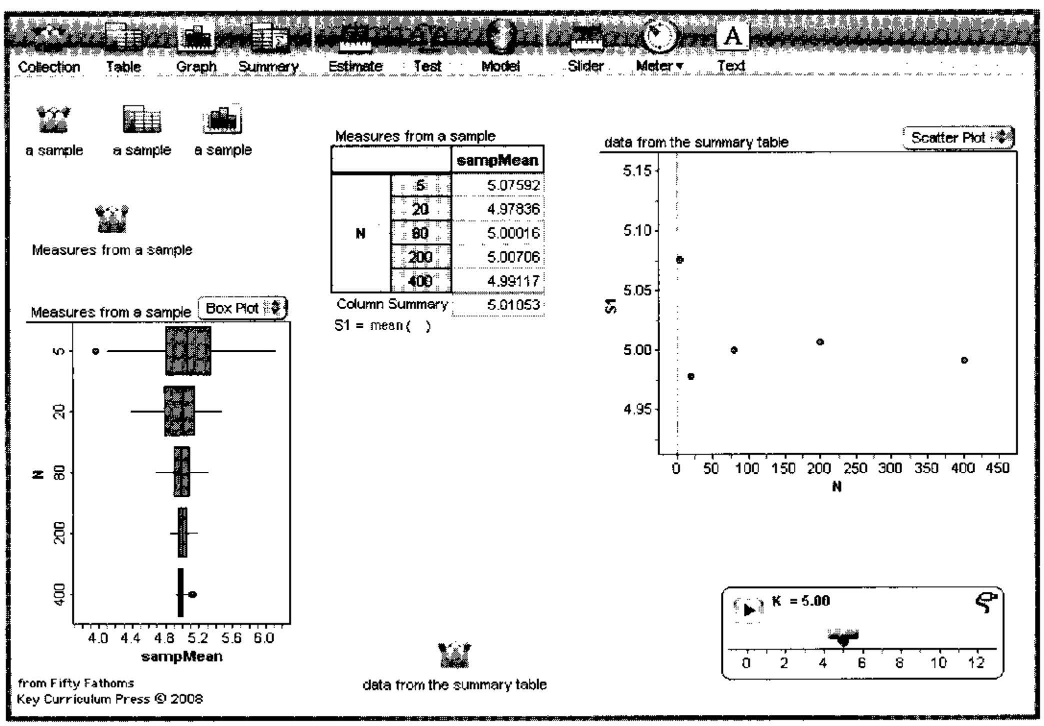
How the width (as measured by IQR) of a sampling distribution of the mean is inversely proportional to the square root of the sample size

This demo follows up on Demo 23, "Sampling Distributions and Sample Size."

By the conclusion of that demo, we had collected sampling distributions of the mean repeatedly for different-sized samples. We saw that the distribution of those sample means got thinner as *n* increased. The question is "How *much* thinner do these distributions get?"

This is important because the width of these distributions parallels the width of the corresponding confidence interval. That is, if we're estimating a statistic such as the mean (or the median, or the standard deviation), we need to predict how good our estimate is likely to be. We know that it will be better the bigger the sample is—*how much* better is what we'll find out. In practical terms, larger samples cost more. Does the extra accuracy legitimize the extra cost?

In this demo, we'll essentially start with the end of the previous demo and extend it.



What To Do

▹ Open **Width of Dist Depends on N.ftm.** It should look like the illustration.

We have "iconified" the objects for the original sample in the upper left, and removed the controls for the mean and standard deviation of the sample—they're fixed at 5.0 and 1.0, respectively. We also moved the "measures" plot to the lower left and changed it from a histogram to a box plot. (You can change it back using the menu in the graph itself, if you wish.) Each box plot displays the means for 100 samples.

The new elements are a *summary table,* currently listing the means of the 100 sample means for each of the different sample sizes; a collection—a box of balls—representing the data in the table itself; and a graph of that data, currently showing the means of the sample means as a function of sample size. As you can see, they're all near 5.0, which is what we would expect.

Now we have to compute and display the spread.

- ▹ Click the summary table once to select it. A **Summary** menu appears in the menu bar.
- ▹ Choose **Add Formula** from the **Summary** menu. The formula editor appears.
- ▹ Enter **iqr( )** (with nothing inside the parentheses). Close the editor with **OK.** A new number—the interquartile range (IQR)—appears in every cell of the summary table, as shown in the illustration.

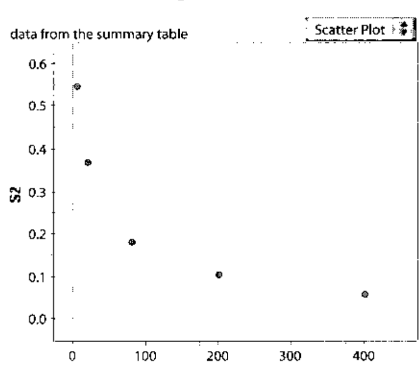
Measures from a sample

		sampMean
N	5	5.07592 0.546265
	20	4.97836 0.370348
	80	5.00016 0.182215
	200	5.00706 0.104025
	400	4.99117 0.0598831
Column Summary		5.01053 0.169316

S1 = mean( )
S2 = iqr( )

You can see by the legend that the second element—called **S2**—is the interquartile range. Note how the IQR gets smaller as **N** gets larger.

- ▹ Now we want to put the IQR on the graph. Double-click the collection—the box at the bottom of the window—called **data from the summary table.** That opens its *inspector*.
- ▹ Drag the label **S2** (that's the IQR of the sample means, remember?) from the inspector to the vertical axis of the graph, replacing **S1** (which is the mean of the sample means).

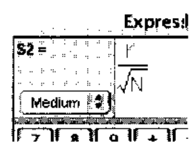


You should see a graph like the one in the illustration. We can see that, indeed, the IQR (**S2**) decreases as n increases, but what is the functional form of the decrease? Suppose we think that an inverse relationship is a good model for the data, that is,

$$IQR(n) = \frac{K}{n}$$

where K is a constant (it's also the name of the slider that we'll use). We'll make this mistake—and then we'll fix it.

- ▹ Click the graph to select it. Then choose **Plot Function** from the **Graph** menu. The formula editor appears.
- ▹ Enter **K / n.** Close the editor with **OK.** A function appears.
- ▹ Use the slider named **K** to make the function fit as well as you can. You'll see that no value of **K** makes the curve fit well—it must be the wrong function.
- ▹ Double-click the function (**S2 = K / n**) at the bottom of the graph to edit the function.
- ▹ Change it to be $K / \sqrt{n}$, as shown in the illustration.



You can use the square root key in the editor to get the radical sign. Close the editor with **OK.**

- ▹ Again, use the slider **K** to fit the curve. It doesn't fit perfectly, but it does a lot better than the "straight" inverse did.

Extensions

Let's see what happens when we use a different measure of spread. We'll use sample standard deviation instead of interquartile range.

- ▹ Close the inspector.
- ▹ At the bottom of the summary table, double-click where the formula reads **S2 = iqr( )**. In the formula editor, enter **s( )**, which is the sample standard deviation. Now the graph of **S2** plots **s( )** instead of **iqr( )**. Use the slider to fit the function to the data.
- ▹ You get a different value for **K**. Larger or smaller? Why?
- ▹ As before, we have also collected **sampMedian** and **sampMax** in the measures collection. You can put those in the summary table and explore how their means and spreads change with sample size.

Challenges

1 Use the **percentile( )** function to compute a 90% band instead of IQR, and re-collect the measures to get the graph. Does this fit a curve better? Worse? About the same?

2 What does **K** really mean (especially when you use SD instead of IQR)? Why does the point at **n = 5** not fit the curve as well?

3 Suppose you didn't think of using the square root in the denominator—you just saw that the inverse function didn't work. What could you do, in exploring the data, to "discover" the square root relationship? **Sol**

What You Should Take Away

The point of all this is to become more familiar with the properties of sampling distributions. Here, we see that for the mean at least, the spread of its sampling distribution decreases more or less as the square root of the sample size—no matter what we use to measure spread.

It also helps to see that we can treat results from statistical simulations as data, and we can treat them the same way as we would if we were modeling a physical phenomenon with a function.

Theory Corner

We discuss why this works from a theoretical standpoint in "The Distribution of the Sample Mean" in Appendix B.