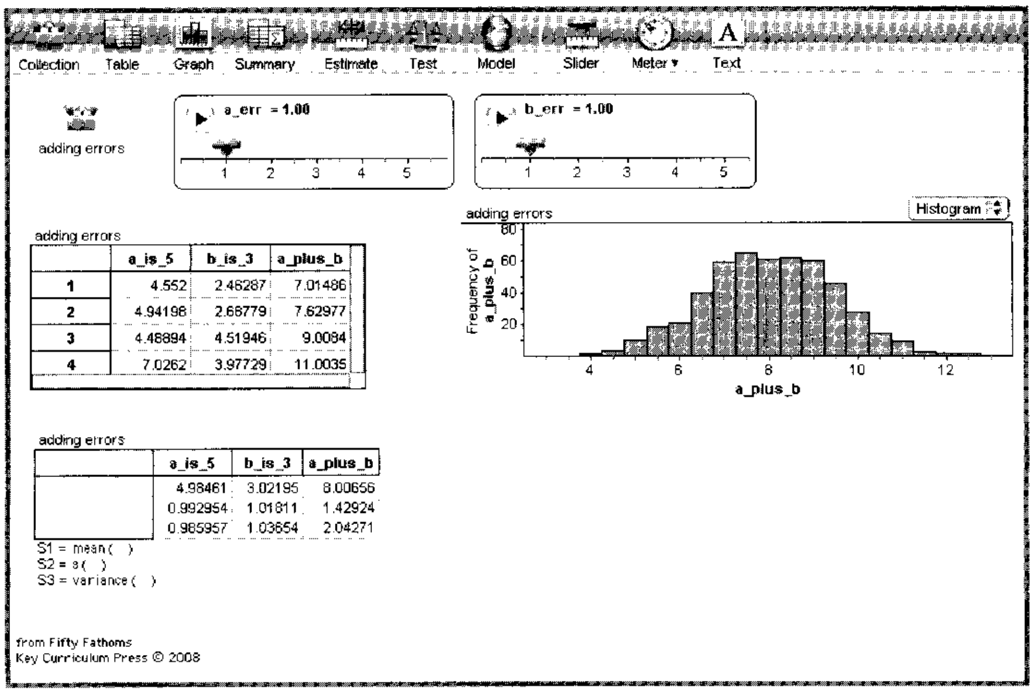
Demo 22: How Errors Add

Basic error analysis • How to find the error in the sum of two quantities that each have some measurement error

It is a curious thing that when you add two (independent, normally distributed) random variables, their means add in the usual way, but you take the Pythagorean sum of their standard deviations to get the standard deviation of the sum. That is,

$$\sigma_{tot} = \sqrt{\sigma_1^2 + \sigma_2^2}$$

This demo lets you see that happen.



What To Do

▹ Open **Adding Errors.ftm.** It will look something like the illustration.

The collection **adding errors** contains the data you can see in the case table below it: values for **a_is_5**, **b_is_3**, and their sum, **a_plus_b.** The attribute **a_is_5** has the value 5, plus a (possibly negative) random error whose standard deviation is controlled by the slider **a_error.** Analogously for **b_is_3.** Below the case table is a summary table where you can see the means, standard deviations, and variances of those three attributes. The distribution of the sum appears in the graph.

▹ Choose **Rerandomize** from the **Collection** menu. Note in the top row of the summary table that the means seem to add (the means of **a** and **b**—5 and 3—add to 8, which is about the mean of the sum), but that the standard deviations—the middle row of the summary table—do not (since $1 + 1 \neq 1.4$).

⇨ The shortcut for **Rerandomize** is **⌘+Y** on the Mac or **Control+Y** in Windows.

▹ Move the sliders **a_error** and **b_error** to see how they affect the graph and the numbers in the summary table.

▹ Verify that nothing you do changes the means very much.

- ▷ Verify that if **a_error** is large compared to **b_error,** then the standard deviation of **a_is_5** is about the same as the standard deviation of **a_plus_b.**
- ▷ Verify that if **a_error** is about the same as **b_error**, the standard deviation of **a_plus_b** is about the same as 1.4 times **a_error.**
- ▷ Verify that, though the standard deviations do not add in the obvious way, the *variances*—the numbers in the bottom row of the summary table—seem to.
- ▷ Verify that if you set one error to 3 and the other error to 4, the error of the sum is about 5. (This is just like a 3-4-5 triangle. The variances should be about, but not exactly, 9, 16, and 25.)

If that's true, then we can add the mean squares of the errors to get the mean square error of the sum. That is,

$$SD_a^2 + SD_b^2 = SD_{total}^2 \quad \text{or}$$

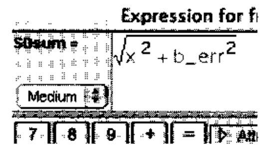
$$SD_{total} = \sqrt{SD_a^2 + SD_b^2}$$

Let's look at the data again, recording what we find out and comparing it to that conjecture.

- ▷ Return the two error sliders to 1.0.
- ▷ Choose **Show Hidden Objects** from the **Object** menu. A measures collection and a graph appear.
- ▷ Double-click the measures collection to open its inspector. Tell it to **Re-collect measures when source changes** by clicking in that checkbox. Close the inspector.
- ▷ Now drag the **a_error** slider and see what happens. Be sure to go both above and below 1. The graph will update to show you how the standard deviation of the sum **SDsum** changes as a function of the standard deviation of **a_is_5,** which is **SDa.**

Let's put in the function.

- ▷ Click once on the scatter plot to select it, then choose **Plot Function** from the **Graph** menu. The formula editor opens.
- ▷ Enter the formula $\sqrt{x^2 + b\_err^2}$, as shown in the illustration.



- ▷ Press **OK** to close the editor. Notice how well the function models the points.
- ▷ Set **b_error** to 3, and then leave it there as you change **a_error** again.

Challenges

1 It looks as if the function is pretty straight if you get far from the vertical axis. Find the equation for that line and explain why it has to be that way—using reasoning about errors. **Sol**

2 The function looks flat as you get close to the vertical axis. Explain that too. **Sol**

Theory Corner

You can read about why variances add in the section "Variance" in Appendix B.