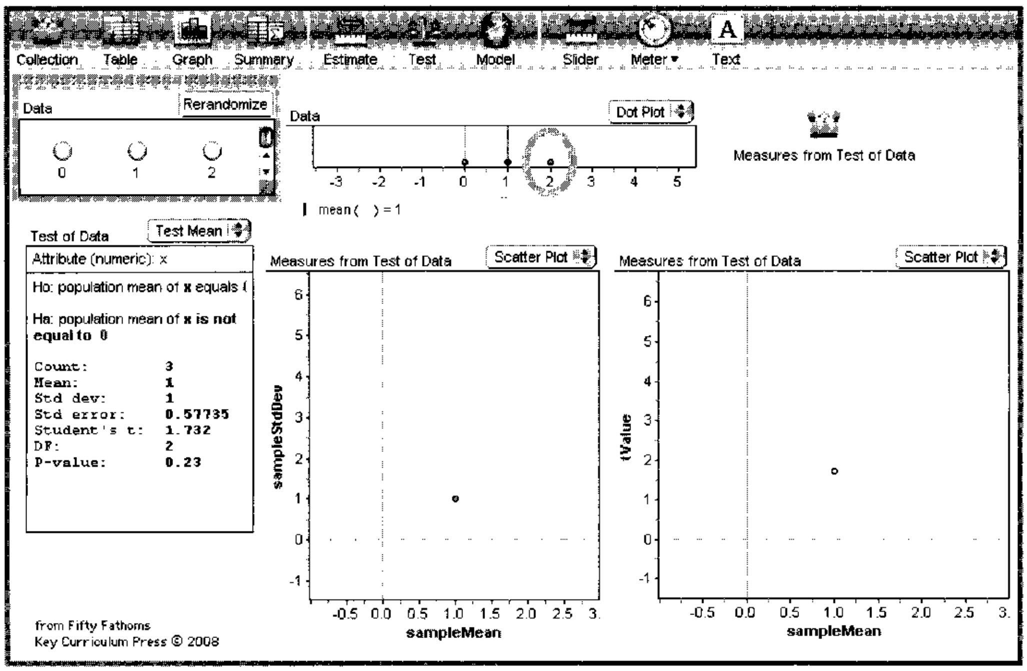
Demo 19: A Close Look at the *t*-Statistic

How sample mean, standard deviation, t, and P interrelate • How they depend on the values of individual points in a sample

In this demo, we'll test a sample of three points. We'll use the *t*-test of the mean to see if we can distinguish that mean from zero. In fact, we won't really care about the results of the test. Rather, we'll be looking at data from many tests to learn how the center and spread of the data relate to the *t*-statistic and to its associated *P*-value.



What To Do

▹ Open **Close Look at Student's t.ftm**. It should look something like the illustration.

This document has the original data at the upper left, the *t*-test below it, a graph of the data top center, and two big graphs in the middle that display the results of the test. As we change the data, Fathom will add points to the graph as fast as it can. (Fathom collects these test results in the collection, upper right; the graphs are of data from that collection.)

Observe how the two graphs display the sample mean, standard deviation, and *t*-value in the test that you can see. (If the test is unfamiliar and you'd like a more complete explanation, select it and uncheck **Verbose** from the **Test** menu; you'll need to shrink it later to make enough screen space to see both the test and the graphs.)

▹ In the short, wide graph at the top, grab the right-hand point—the one at a value of 2, indicated in the illustration—and drag it to the right. See how the mean (**sampleMean**) and standard deviation (**sampleStdDev**) both increase in the two graphs below, but that **tValue** decreases.

▹ Now drag the point all the way to the left, watching the two graphs. Be sure you get points near the maximum or minimum in both graphs. Return the point to near 2 when you're done. (**Undo** is perfect for this.)

Questions

1 Where do you put the "2" point to make the standard deviation a minimum? Why?

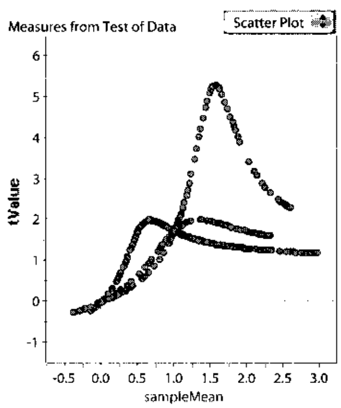
2 When you moved the point to the right, why did t decrease? Isn't the mean getting farther from zero? **Sol**

3 Is the place where t is a maximum different from the point where the standard deviation is a minimum?

Onward!

▹ Now, with the "2" point back where it belongs, drag the "1" point back and forth.

▹ Finally, put the "1" point back and drag the "0" point. The right-hand graph will look something like the one in the illustration.



More Questions

4 Which point do you have to move to get the smallest standard deviation? Why? **Sol**

5 Which point do you have to move to get the largest t?

6 Explain the answer to the previous question in terms of the situation; that is, since it's a test to see if the mean is zero, explain (ideally without formulas) why moving *that* point makes the null hypothesis the *least* believable.

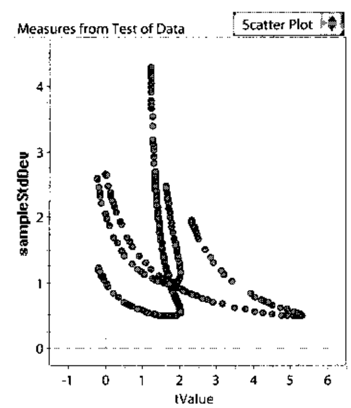
Extension

Suppose we want to explore the P-values. In which of the tests we have done would we reject the null hypothesis $\mu = 0$ at the 5% level?

▹ Double-click the highest point on the **sampleMean** vs. **tValue** graph—the point with the largest value for t. The collection's inspector opens, and you should see the value for P (called **pValue**), which is about 0.03. So we would reject that point.

▹ Drag the name of the attribute **pValue** into the *middle* of that scatter plot. The points color, and a legend appears at the bottom. The dark purple points are for low p and would be rejected.

▹ Let's get a more accurate picture, using the other (left-hand) graph. First, drag the attribute **tValue** to the horizontal axis of the left graph, replacing **sampleMean.** The graph will look like the one in the illustration.

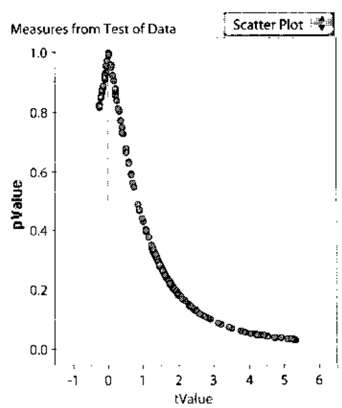


▹ Take a moment to appreciate this graph! It shows how the largest values of t for the situations we have looked at have small standard deviations.

Note that this also suggests an explanation for why the t-distribution is different from the normal: The points in the long tails of the t-distribution are from samples whose means are offset from the true mean, but happened to have a small spread. This often happens with three points when all three are on one side of the mean (it happens one-quarter of the time, after all). They will naturally have a smaller SD than the average sample. A small SD means a small SE and a larger t-statistic. Similarly, a sample with a large SD will be more likely to straddle the mean because it's so fat. But not only is the sample mean closer to the true mean, the t-statistic will be pushed closer still because the difference is measured in units of SEs—and there are fewer SEs between the mean and the population value because of the large spread.

Of course, the more points there are, the less likely you are to get a sample SD that's very extreme—and the closer the distribution gets to normal.

▹ Now drag **pValue** to the vertical axis of that graph, replacing **sampleStdDev.** This graph shows how p depends on t. Large t—positive or negative—means that you get a small p, which is what you need to reject the null hypothesis.



Challenges

7 Zoom in to this last graph to figure out what values of t correspond to values of p less than 0.05. Confirm that they're the same as those in a critical-values table.

8 Add a point to the sample: Select one of the left-hand gold balls, **Copy** it (**Edit** menu), and **Paste** it (**Edit** menu again). The new point will appear in the dot plot, and the t-test will report a sample size of 4 instead of 3. Drag these points. Explain what happens on the t vs. P graph—why isn't it the same as before? How is it different?

9 As in the first task, zoom in to see where the critical values for t are. There are now two—one for a sample of 3, one for 4. Confirm that the one for 4 is smaller than the one for 3, and explain why. **Sol**

10 Select all of the points in the data graph (the short, wide one at the top) by dragging a marquee (dashed rectangle) around them. Then drag them all at once. Describe what happens in the (now messy) graph of **tValue** vs. **sampleMean**, and explain it.