Algebra II Individual

If a question asks for the CLOSEST answer…
… and the correct result is in-between two answers exactly, select EITHER.
… and there are MULTIPLE correct results, select an answer that is closest to ONE of them.
… only select NOTA if you feel the question is DEFECTIVE.

OTHERWISE (if the closest answer is not requested), select NOTA based on the instructions given by the proctor prior to the test.

1) Jana is a complex person. She spends A hours of each day in the real world, and all of the rest (B) dreaming (assume there are exactly 24 hours per day). Jana considers the value of a day to be equal to the absolute value of the complex number $A + Bi$, where $i = \sqrt{-1}$. If Jana considers the value of yesterday to be $\sqrt{386}$, which of the following is closest to the positive difference between A and B on that day? Assume, of course, that neither can be negative.

a. 0 b. 4 c. 9
d. 18 e. NOTA

2) Solve for y: $\dfrac{97-(4\div 6+4)\cdot 9+2y-5+\left(6^{7+5(-1)^3}\right)y}{y+8-3(4+5\cdot 7)+60-3}=1$.

a. 3 b. −3 c. 1
d. −1 e. NOTA

3) If $f(x) = 2-\sqrt{-x+1}$, what is the range of *f(x)* for all x in the domain of *f(x)*?

a. $(-\infty, 2]$ b. $(-\infty, 1]$ c. $[1, \infty)$
d. $[2, \infty)$ e. NOTA

4) What is the description of the locus of points on a plane described by the following: $x^2 + y^2 - 10x + 6y = -34$?

a. Circle b. Ellipse c. Hyperbola
d. Parabola e. NOTA

5) Segment A and Line B are both on the xy-plane. Line A is the line segment drawn from (2,-13) to (7, 0). Line B contains the midpoint of Segment A, and is also perpendicular to Segment A. Find the equation of Line B.

a. $13x - 5y = 91$ b. $13x - 5y = -62$ c. $5x + 13y = 91$

d. $5x + 13y = -62$ e. NOTA

6) If $g(x) = 4\ln x + 1$, the value of $g^{-1}(\ln 81 + 1)$ is closest to which of the following?

a. 11 b. 6 c. 1

d. –4 e. NOTA

7) Find the decimal representation of 11111011000_2. Sum the digits of this result (in base 10, of course). Square the result, and find which of the following is closest to it:

a. 34 b. 68 c. 182

d. 6200 e. NOTA

8) Through which quadrant does the graph of $7x - 2y = 28$ <u>not</u> pass?

a. I b. II c. III

d. IV e. NOTA

9) What is the sum of the integral solutions to $|x - 3| - 4 \geq 0$.

a. 27 b. -27 c. –21

d. Infinite Sum e. NOTA

10) Given that $\frac{6x}{(x+4)^2} = \frac{A}{x+4} + \frac{B}{(x+4)^2}$, what is the value of $A - B$?

a. -18 b. -30 c. 30

d. 18 e. NOTA

11) Who first used the letter e to denote the base of the natural logarithm function?

a. Leonhard Euler b. Albert Einstein c. Eratosthenes

d. Euclid of Alexandria e. NOTA

12) If the remainder of the following quotient:

$$\frac{x^{100} - x^{99} + x^{98} - x^{97} + \ldots + x^2 - x + 1}{x + 1}$$

is written as $\frac{A}{x+1}$, then find the value of A.

a. 1 b. -1 c. –101

d. 101 e. NOTA

13) My four favorite numbers are unique natural numbers (I hear you breathe a sigh of relief – but don't be fooled, this question is anything but natural). If you add them all and divide by the sum of the lowest two, you'll get 3.75. The largest one is equal to the sum of the second and third largest. You'll find they're all under 10, and only the second largest is divisible by two. Which of these numbers is closest to the product of my numbers?

a. 70 b. 80 c. 90

b. 100 e. NOTA

14) My three least favorite numbers are consecutive integers. The sum of the squares of these numbers minus their sum is 20. Most of the numbers are odd. Which is closest to their product?

a. –10 b. –4 c. 0

d. 5 e. NOTA

15) My favorite hyperbola has the equation $\frac{x^2}{e^\pi} - \frac{y^2}{\pi^e} = 1$. What is the sum of the slopes of the asymptotes of this hyperbola?

a. $e^\pi - \pi^e$ b. $\pi^e - e^\pi$ c. $e^\pi + \pi^e$

d. 1 e. NOTA

16) Let $f(x) = Ax^2 + Bx + C$. The discriminant of $f(x)$ is 49, and the product of the roots of $f(x)$ is twice the sum of the roots of $f(x)$. Determine the value of A in terms of B.

a. $\frac{B^2 - 49}{8B}$ b. $\frac{B^2 - 49}{8}$ c. $\frac{B^2 - 7}{8}$

d. Cannot be determined e. NOTA

17) Which expression is equivalent to $\dfrac{xy^2z + x^{-2}y}{y^{-1}z^3}$ for $xy \neq 0$?

a. $\dfrac{y^2\left(x^3yz+1\right)}{x^2z^3}$ b. $\dfrac{xy^2z\left(y-z\right)}{y^2-z}$ c. $\dfrac{y^2\left(x^3z+y\right)}{xz^3-1}$

d. $\dfrac{x^3y}{x^2z+1}$ e. NOTA

18) $M = \begin{vmatrix} x & 2 \\ 2x+4 & 2x-4 \end{vmatrix}$. If the determinant of M is equal to the smallest prime number, in which of the following intervals do the possible values of x fall?

a. [-3,2] b. [-2,3] c. [-1,4]
d. [0,5] e. NOTA

19) Given that $\sqrt{x+\sqrt{x+\sqrt{x+...}}} = n$ and $4n = 28 - x$, which of the following is closest to the value of $\dfrac{n}{x}$, if n and x are real numbers?

a. 1 b. $\dfrac{1}{2}$ c. $\dfrac{1}{4}$

d. $\dfrac{1}{6}$ e. NOTA

20) If the center of the circle that passes through the points $(-3,1)$, $(4,0)$, and $(6,4)$ is (h,k), the value of $h+k$ is closest to which of the following?

a. 6 b. 2 c. -2
d. –6 e. NOTA

21) Given $f(x) = \dfrac{x+3}{x^2-1}$ and $g(x) = \dfrac{1}{\sqrt{x}-1}$. What is the product of the whole numbers that are excluded from the domain of $(f \circ g)(x)$?

a. 0 b. 1 c. 2
d. 4 e. NOTA

22) Travis is buying pizzas from a local pizza delivery shop. The shop charges a delivery fee of 5% of the total order. One-topping pizzas are $10, and two-topping pizzas are $15. He just ordered seven pizzas for a total of $94.50 (Travis lives in a place with no tax or other fees). His friends eat half of the two-topping pizzas, and then half of all the remaining pizzas. Each pizza has 8 slices. Travis decides to sell the remaining pizza for $1 per slice. Which of the following is closest to how much money he makes from these sales?

a. $18	b. $19	c. $20
d. $21	e. NOTA	

23) Which is closest to the value of $\dfrac{3\left(3\left(3^{2007}+3^{2006}\right)-3^{2005}\right)}{3^{2005}+3^{2004}}$?

a. 100	b. 200	c. 300
d. 400	e. NOTA	

24) $a \oplus b = (b-2)\dfrac{a \oplus (b-2)}{2}$. Given that $5 \oplus 1 = \ln 2$, find the value of $5 \oplus 7$.

a. $(3\ln 2)-2$	b. $4\ln 4$	c. $3\ln 8$
d. $\ln 64$	e. NOTA	

25) A quartic function has the form $f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$ with A>0. If *f(x)* has 4 complex roots, which of the following must be true regarding *f(x)*?

I. $f(x) \geq E$ for all $x \in \Re$
II. f(x) never crosses the x-axis for all $x \in \Re$
III. B=0

a. I only	b. II only	c. II and III only
d. I and III only	e. NOTA	

26) If $\log\left(\sqrt[3]{25}\right)=A$ and $\log\left(\frac{1}{49}\right)=B$, what is the value of $\log\left(5\sqrt{35}\right)$ in terms of A and B?

a. $\frac{3A-B}{2}$ b. $\frac{9A+B}{4}$ c. $\frac{9A-B}{4}$
d. $\frac{3A+B}{2}$ e. NOTA

27) What is the sum of the real solutions of the following equation: $2x\sqrt{x}-20\sqrt{x}-6x=0$?

a. 3 b. 5 c. 25
d. 29 e. NOTA

28) The maximum y-value of an ellipse occurs at (3,4), the minimum y-value of the same ellipse occurs at (3,-8), and the length of the semi-major axis is 10. The distance between the foci of this ellipse is closest to which of the following?

a. 5 b. 10 c. 15
d. 20 e. NOTA

29) Given that $f(x)=2-x-i$ and $g(x)=2-(4-x)i$ where $i=\sqrt{-1}$. Find which is closest to the value of $\left|\frac{f(1)}{g(1)}\right|\cdot\sqrt{5\cdot f(-i)+3}$?

a. $-2\sqrt{2}$ b. $2\sqrt{2}$ c. $-3\sqrt{2}$
d. $3\sqrt{2}$ e. NOTA

30) Okay, here's an easy one. How many digits are to the left of the decimal point in $\sqrt[3]{2008}$?

a. 1 b. 2 c. 3
d. 4 e. NOTA

Question 1

A) The resistance of copper wire R varies directly as its length (L). Find the value of the constant of variation in terms of L and R.

B) Suppose x varies jointly as y and z. Find the value of y when $x=-2$ and $z=25$, if $y=210$ when $x=-7$ and $z=-10$.

C) The volume of a gas varies directly as its temperature and inversely as its pressure. Helga is working with a gas in a laboratory at Subway University. At a temperature of 110 Kelvin and a pressure of 15 pascals, she measured the volume of this gas as 33 cubic centimeters. What volume, in cubic centimeters, should she expect if she increases the temperature to 180 Kelvin and increases the pressure to 28 pascals?

D) z is directly proportional to the square root of x and inversely proportional to the cube of y. Given that $x+y=2$, $x+z=0$, and $2y+z=-1$, find the value of the constant of variation.

Question 2

A) What is the shortest distance from the point (9,5) to the circle defined by $x^2 + y^2 - 2x - 2y - 2 = 0$?

B) What is the area of the circle defined by $x^2 + y^2 = 22$ enclosed in Quadrant III?

C) What is the area of the ellipse defined by $16x^2 + 9y^2 - 64x - 80 = 0$?

D) What is the distance from a focus to the point with the largest x-value for the ellipse defined by $9x^2 + y^2 + 54x - 2y + 73 = 0$?

Question 3

A. What is the sum of the values of x if $|2x - 3| = |-7x + 5|$?
B. If $|x^2 + 7x + 14| = 1$ is solved over the complex numbers, what is the product of all possible values of x?
C. How many integral solutions does the following system have:

$$|3x+5| \geq 11$$
$$|-2x-6| < 2$$

D. How many of the following are one-to-one functions:
   I. $f(x)=|7|$
   II. $g(x)=|74x-192|$
   III. $z(x)=|x^2+3x+2|$
   IV. $w(x)=|x^3-3x+7|$
   V. $€(x)=|x^4|$

Question 4

A) Find the equation of the line through the points (3,3) and (-2,7) in the form Ax+By=C, where A, B, and C are relatively prime integers and A>0.
B) Find the equation of the line perpendicular to the line through the points (5,-2) and (-2,-1) where the two lines share the same x-intercept, in the form Ax+By=C, where A, B, and C are relatively prime integers and A>0.
C) Find the shortest distance between the point (0,2) and 2x+4y=3.
D) Find the shortest distance between 2x-y=5 and 2x-y=3.

Question 5

A) If $f(x) = -2x^2 + mx - 7$ has exactly one real root, what is the product of the possible values of m?
B) What is the minimum value of $g(x) = 3x^2 - 5x + 6$?
C) A quadratic in the form $y = ax^2 + bx + c$ has roots {-6,4} and has a discriminant of 4. If a,b>0 and c<0, what is the sum of a,b and c?
D) How many possible values are there for the rational roots of h, by the Rational Root Theorem, for the function $h(x) = 6x^2 - kx + 4$, where k is an integer?

Question 6

For the following questions, $i = \sqrt{-1}$

A) $(2i + 1)^6 = a + bi$ where a and b are real numbers. What is a+b?
B) What is the value of i^{214567}?
C) One of the roots of the function $f(x) = x^4 - 3x^3 + 7x^2 - 8x + 6$ is $(1+i)$. What is the sum of the other three roots?
D) If $\frac{6+i}{5-2i} = a + bi$, what is the value of a+b?

Question 7

A) Solve for x: $\sqrt{-4x+\sqrt{-4x+\sqrt{-4x+...}}}=x+2$

B) If $\frac{7}{6x^2+13x+6}=\frac{A}{2x+3}+\frac{B}{3x+2}$, what is A+B?

C) Solve for x: $\frac{1}{\frac{1}{x}+\frac{1}{x+3}}-\frac{1}{1+\frac{1}{x}}=\frac{3}{28}$

D) Solve for x: $\sqrt{x+5}-\sqrt{2x}=0$

Question 8

A) Given a=ln2, b=ln3, and c=ln5, find ln(14400) in terms on a, b, and c.
B) Solve for x over the real numbers: $2^{2x}+2^{x}-6=0$.
C) We are going to invest \$1.00 in an account that earns interest for 24 months. Determine how much money will be in the account if interest is compounded annually at an interest rate of 4%, to the nearest cent.
D) We are going to invest \$1.00 in an account that earns interest for 2400 months. Determine how much money will be in the account if interest is compounded continuously at an interest rate of ln(50)% to the nearest cent.

Question 9

A) What is the length of the latus rectum of the parabola defined by $y=2x^2+3x-7$?

B) A parabola is in the form $f(x) = ax^2 + bx + c$, where $f(-1) = 18$, $f(1) = 6$ and $f(2) = 15$. What is $a+b+c$?

C) What is the distance between the foci of the hyperbola defined by $4x^2 - 9y^2 - 32x - 18y + 19 = 0$?

D) What is the point of intersection of the asymptotes of the hyperbola defined by $36x^2 - y^2 + 360x + 6y - 855 = 0$?

Question 10

A) What is the prime factorization of 2008?

B) How many distinct, positive, integral factors does 6600 have?

C) How many zeroes can be found at the end of (173!) ?

D) $27 \bmod 12+13 \bmod 5-87 \bmod 9 \equiv A \bmod 6$. Solve for A, where A is smallest possible natural number that satisfies the equation.

Question 11

A) Solve for x and y in the following system and write your solution as (x,y):

$$4x + 6y = 12 \quad \text{AND} \quad -7x + 3y = -39$$

B) Solve for x and y in the following system and write your solution as (x,y):

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{6} \quad \text{AND} \quad \frac{3}{x} + \frac{2}{y} = \frac{13}{6}$$

C) Solve for x and y in the following system and write your solution as (x,y) if x,y>0:

$$x^2 - y^2 = 25 \quad \text{AND} \quad x^2 + 2xy + y^2 = 625$$

D) What is the area that is enclosed by the solution set to the following system:

$$y \le 3x + 7$$
$$y \le -2x + 2$$
$$y \ge 0$$

Question 12

A) Evaluate: $\begin{vmatrix} 1 & 4 & 0 \\ -3 & 7 & 5 \\ 2 & -4 & -3 \end{vmatrix}$

B) What value of m makes matrix A not invertible if A= $\begin{bmatrix} 1 & m \\ 12 & -6 \end{bmatrix}$?

C) If the determinant of matrix D is 4 and the determinant of matrix F is -3, what is the determinant of 2D – 4F?

D) When solving a system of equations of 2 variables (x and y), Paige uses Cramer's Rule. For the value of x she gets $\dfrac{\begin{vmatrix} 2 & -3 \\ -3 & 1 \end{vmatrix}}{\begin{vmatrix} 12 & -3 \\ 11 & 1 \end{vmatrix}}$. Find the value of x+y.

Question 13

Please answer the following questions as TRUE or FALSE. Write out the WHOLE WORD true or false. No credit will be given to answers of T or F.

A) Functions of the form $f(x)=ax^{2n-1}$ are considered to be one-to-one functions for real numbers a, and natural numbers, n.
B) Conic sections are formed by the intersection of a plane and a double-napped cone of infinite height.
C) $2+7\times3-10\div5\times6+3-1-13=0$.
D) Polynomial functions of degree *n* sometimes do not have *n* real roots.

Question 14

A) $1600_7=x_{10}$. Solve for x.
B) $1600_{12}=x_{10}$. Solve for x.
C) $1600_{10}=x_7$. Solve for x.
D) $1600_{10}=x_{12}$. Solve for x.

Question 15

A) Expand and simplify: $(2x+1)\left(x-\frac{1}{2}\right)^2$.

B) The remainder when $6x^4-2x^3+3x-1$ is divided by x^2+3 is $\frac{R(x)}{x^2+3}$.

What is the expression for $R(x)$?

C) For how many distinct values of x does $y=x^4-4x^3+5x^2-4x+4$ intersect with the x-axis?

D) Find the domain of $f(x)=\frac{x+1}{x^2+4x}$ in interval notation.

Algebra II Indiv Answers

1. D
2. E
3. A
4. E
5. D
6. C
7. B
8. B
9. C
10. C
11. A
12. D
13. B
14. B
15. E
16. A
17. A
18. E
19. C
20. A
21. A
22. C
23. A
24. D
25. B
26. C
27. C
28. C
29. B
30. B

Algebra II Team Answers

1. a. $\frac{R}{L}$
   b. -24
   c. $\frac{405}{14}$
   d. $-9\sqrt{15}$

2. a. $4\sqrt{5}-2$
   b. $\frac{11\pi}{2}$
   c. 12π
   d. 3

3. a. $\frac{58}{45}$
   b. 195
   c. 0
   d. 0

4. a. 4x+5y=27
   b. 7x-y=-63
   c. $\frac{\sqrt{5}}{2}$
   d. $\frac{2\sqrt{5}}{5}$

5. a. -56
   b. $\frac{47}{12}$
   c. $\frac{-21}{5}$
   d. 16

6. a. 161
b. $-i$ or $-\sqrt{-1}$
c. $2-i$
d. $\frac{45}{29}$

7. a. $\frac{\sqrt{41}-7}{2}$
b. $\frac{7}{5}$
c. 3
d. 5

8. a. 6a+2b+2c
b. 1
c. 1.08
d. 2500

9. a. $\frac{1}{2}$
b. 6
c. $2\sqrt{13}$
d. (-5,3)

10. a. $2^3 \cdot 251$
b. 48
c. 41
d. 6

11. a. $\left(5,\frac{-4}{3}\right)$
b. (2,3)
c. (13,12)
d. $\frac{20}{3}$

12. a. 3
b. $\frac{-1}{2}$
c. 20
d. $\frac{-13}{9}$

13. a. true
b. true
c. true
d. true

14. a. 637
b. 2592
c. 4444
d. B14

15. a. $2x^3 - x^2 - \frac{1}{2}x + \frac{1}{4}$
b. 9x+53
c. 1
d. $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$

Algebra II Individual Solutions

1. Since the day is made up entire of A and B we can determine that $A+B=24$. Now also $|A+Bi|=\sqrt{A^2+B^2}=(\sqrt{386})^2 \Rightarrow A^2+B^2=386$. Now notice by substitution that
$A^2+(24-A)^2=386 \Rightarrow 2A^2-48A+576=386 \Rightarrow A^2-24A+95=0 \Rightarrow A=5,19$.
A can be either 5 or 19, but substituting these values back into the original equation gives the values of B as 5 or 19, so we just need to know that one is 5 while one is 19. Thus the positive difference is 14 which is closest to 18.

2.
$$\frac{97-(4\div 6+4)\cdot 9+2y-5+\left(6^{7+5(-1)^3}\right)y}{y+8-3(4+5\cdot 7)+60-3}=1$$
$$\frac{97-42+2y-5+36y}{y+8-117+60-3}=1$$
$$50+38y=y-52$$
$$37y=-102$$
$$y=\frac{-102}{37}$$

3. The range of $\sqrt{-x+1}$ is $[0,\infty)$. The range of $-\sqrt{-x+1}$ is $(-\infty,0]$. So the range of $2-\sqrt{-x+1}$ is $(-\infty,2]$.

4. Completing the square yields
$(x^2-10x+25)+(y^2+6y+9)=-34+25+9 \Rightarrow (x-5)^2+(y+3)^2=0$. This is the equation of a point, thus E.

5. The midpoint of the segment is $(\frac{2+7}{2},\frac{-13+0}{2})\equiv(\frac{9}{2},\frac{-13}{2})$. The slope of the segment is $\frac{-13-0}{2-7}=\frac{-13}{-5}=\frac{13}{5}$. So the slope of the line perpendicular is $\frac{-5}{13}$. Thus, the line is
$(y-(\frac{-13}{2}))=\frac{-5}{13}(x-\frac{9}{2}) \Rightarrow 13y+\frac{169}{2}=-5x+\frac{45}{2} \Rightarrow 5x+13y=-62$

6.

$$g(x) = 4\ln x + 1$$
$$x = 4\ln g^{-1}(x) + 1$$
$$\frac{x-1}{4} = \ln g^{-1}(x)$$
$$g^{-1}(x) = e^{\frac{x-1}{4}}$$
$$g^{-1}(\ln 81 + 1) = e^{\frac{\ln 81 + 1 - 1}{4}}$$
$$g^{-1}(\ln 81 + 1) = e^{\frac{\ln 81}{4}}$$
$$g^{-1}(\ln 81 + 1) = e^{\ln 3} = 3$$

7.

$$11111011000_2 = [1(2^{10}) + 1(2^9) + 1(2^8) + 1(2^7) + 1(2^6) + 0(2^5) + 1(2^4) + 1(2^3) + 0 + 0 + 0]_{10}$$
$$2008_{10}$$

Summing the digits of the results gives 10 and squaring this is 100 which is closest to 68.

8. The line has positive slope, with x-intercept (2,0) and y-intercept (0,-7). Drawing this out shows that the line passes through only quadrants I, III and IV.

9. This inequality is equivalent to $|x-3| \geq 4$. This means $x - 3 \geq 4 \Rightarrow x \geq 7$ or $x - 3 \leq -4 \Rightarrow x \leq -1$. So the integral solutions that work are (7,8,9,10,…) or (-1,-2,-3,-4,…). Now looking at the sum

$$(-1 + -2 + -3 + -4 + -5 + -6 + -7 + -8 + -9 + -10 + \ldots) + (7 + 8 + 9 + 10) \Rightarrow$$
$$(-1 + -2 + -3 + -4 + -5 + -6) + (-7 + 7) + (-8 + 8) + (-9 + 9) + \ldots \Rightarrow$$
$$(-21) + 0 + 0 + 0 + \ldots = -21$$

10. $\frac{6x}{(x+4)^2} = \frac{A(x+4)+B}{(x+4)^2}$. So 6x=Ax+4A+B. So A=6. Then 4(6)+B=0. So B=-24. So 6-(-24) = 30

11. Leonhard Euler

12. To find the remainder we can substitute $x+1=0 \Rightarrow x=-1$ into the numerator of the function. So the remainder is
$(-1)^{100}-(-1)^{99}+(-1)^{98}-(-1)^{97}+\ldots+(-1)^{2}-(-1)+1=1+1+1+\ldots+1+1+1=101$

13. Let the digits (in ascending order) by *A*, B, *C*, D. Using logic notice that *C* is the only digit that can be divisible by 2 which means *C* can be 2,4,6 or 8. Using logic, *C* cannot be 2, since *A* and B are both less than *C*. Trying *C*=4, means that B=3 and *A*=1, so D must be 7 since D=B+*C*. Now, $\frac{A+B+C+D}{A+B}=3.75 \Rightarrow \frac{1+3+4+7}{1+3}=3.75$. So these are the values, and the product is 1(3)(4)(7) = 84 which is closest to 80.

14. The consecutive integers are n, n+1 and n+2. So $n^2+(n+1)^2+(n+2)^2-(n+n+1+n+2)=20 \Rightarrow 3n^2+3n-18=0 \Rightarrow n=-3,2$. By the stipulation that most numbers are odd, n=-3 which means the 3 integers are -3, -2 and -1. So the product is -6 which is closest to -4.

15. The slopes of any hyperbola are exact opposites of one another. Thus the sum of the slopes is 0.

16. The product of the roots can be written as $\frac{C}{A}$ and the sum of the roots can be written as $\frac{-B}{A}$. So, $\frac{C}{A}=2(\frac{-B}{A})$ Now, the discriminant is
$B^2-4AC=49 \Rightarrow 1-\frac{4AC}{B^2}=\frac{49}{B^2} \Rightarrow 1-4(\frac{A}{B})(\frac{C}{B})=\frac{49}{B^2} \Rightarrow 1-4(\frac{A}{B})(\frac{C}{A})(\frac{A}{B})=\frac{49}{B^2}$
$\Rightarrow \frac{B^2-49}{B^2}=-4(\frac{A}{B})^2(\frac{C}{A}) \Rightarrow \frac{B^2-49}{4B^2}=-(\frac{A}{B})^2(\frac{-2B}{A}) \Rightarrow \frac{B^2-49}{4B^2}=\frac{2A}{B} \Rightarrow A=\frac{B^2-49}{8B}$

17.

$$\frac{xy^2z + x^{-2}y}{y^{-1}z^3}$$

$$\frac{xy^2z + \frac{y}{x^2}}{\frac{z^3}{y}}$$

$$\frac{x^3y^2z + y}{x^2} \cdot \frac{y}{z^3}$$

$$\frac{x^3y^3z + y^2}{x^2z^3}$$

$$\frac{y^2\left(x^3yz + 1\right)}{x^2z^3}$$

18. Det(M) = $x(2x-4)-2(2x+4) = 2x^2-8x-8$. Now the smallest prime is 2. So det(M)=2. So $2x^2-8x-8 = 2 \Rightarrow 2x^2-8x-10 = 0 \Rightarrow x^2-4x-5 = 0 \Rightarrow x = -1{,}5$. So none of the given intervals contain both answers, thus E.

19. $\sqrt{x+\sqrt{x+\ldots}} = n \Rightarrow \sqrt{x+n} = n$. Now also $4n = 28 - x \Rightarrow x = 28 - 4n$. By substitution $\sqrt{28-4n+n} = n \Rightarrow 28-3n = n^2 \Rightarrow n^2+3n-28 = 0 \Rightarrow n = -7{,}4$. Now n must be 4 since it is the sum of square roots. From this information, and substitution we find that x = 12. So $\frac{n}{x} = \frac{4}{12} = \frac{1}{3}$ which is closest to $\frac{1}{4}$.

20.

$x^2+y^2+Cx+Dy=E$

$3C-D+E=10$

$6C+4D-E=-52$

$4C+8D-E=-80$

$9C+3D=-42 \to 3C+D=-14$

$7C+7D=-70 \to C+D=-10$

$C=-2$

$D=-8$

$E=8$

$x^2-2x+y^2-8y=8$

$x^2-2x+1+y^2-8y+16=8+1+16$

$(x-1)^2+(y-4)^2=25$

21. $(f\circ g)(x)=\dfrac{\frac{1}{\sqrt{x}-1}+3}{(\frac{1}{\sqrt{x}-1})^2-1}$. There are two problems that must be looked at $\sqrt{x}-1\neq 0 \Rightarrow x\neq 1$. Also, $(\frac{1}{\sqrt{x}-1})^2-1\neq 0 \Rightarrow \frac{1}{\sqrt{x}-1}\neq \pm 1$. Solving for x, yields that $x\neq 0,4$. So the product is (0)(1)(4)=0

22.

$o+t=7 \Rightarrow t=7-o$

$10o+15t+0.05(10o+15t)=94.50 \Rightarrow 10o+15(7-o)+0.05(10o+15(7-o))=94.50$

$\Rightarrow o=3 \Rightarrow t=7-3=4$

Now, since the friends eat half of the two-topping pizzas, 2 two-toppings are left and 3 one-toppings. They then eat half of the remaining pizzas. So there are 1.5 one topping and 1 two topping for a total of 20 slices left over. So he makes a sale of $20.

23. Multiplying we get $\dfrac{3^{2009}+3^{2008}-3^{2006}}{3^{2005}+3^{2004}}$. Dividing numerator and denominator by 3^{2004} we get $\dfrac{3^5+3^4-3^2}{3+1}=78.75$ which is closest to 100.

24.

$$5\oplus 3=(3-1)\frac{5\oplus 1}{2}=\ln 2$$

$$5\oplus 5=(5-1)\frac{5\oplus 3}{2}=2\ln 2$$

$$5\oplus 7=(7-1)\frac{5\oplus 5}{2}=6\ln 2=\ln 64$$

25. A is false because the graph could increase down past the y-intercept. B is true since there are no real solutions. C is false because the sum of the roots will be the sum of the real parts of the complex solutions and if b=0 that implies the sum of the roots is 0.

26. First, notice that $\frac{1}{3}\log(25)=A\Rightarrow 2\log(5)=3A\Rightarrow \log(5)=\frac{3A}{2}$, and $-2\log(7)=B\Rightarrow \log(7)=\frac{-B}{2}$. Now,

$$\log(5\sqrt{35})=\log(5\sqrt{5}\sqrt{7})=\log(5^{3/2})+\log(7^{1/2})=\frac{3}{2}\log(5)+\frac{1}{2}\log(7)=\frac{3}{2}(\frac{3A}{2})+\frac{1}{2}(\frac{-B}{2})$$

$$=\frac{9A-B}{4}$$

27. Rewrite as $2\sqrt{x^3}-6x-20\sqrt{x}=0$. Making the substitution $u=\sqrt{x}$ yields, $2u^3-6u^2-20u=0$. Solving for u yields: $u^3-3u^2-10u=0\Rightarrow u(u^2-3u-10)=0\Rightarrow u(u-5)(u+2)=0\Rightarrow u=0,5,-2$. Substituting these values back into the initial substitution we get that x=0 and 25. U= -2 does not yield a value of x. Therefore the sum of the solutions is 25.

28. First, notice that the length of the semi-minor axis is $\frac{4-(-8)}{2}=6$. Now, let c be the distance from the center of the ellipse to one of the foci, then $10^2=6^2+c^2\Rightarrow c=8$. So the distance between the 2 foci is 2(8)=16 which is closest to 15.

29.First, $\sqrt{f(-i)*5+3}=\sqrt{2*5+3}=\sqrt{13}$. Next, $|\frac{f(1)}{g(1)}|=|\frac{1-i}{2-3i}\bullet\frac{2+3i}{2+3i}|=|\frac{5+i}{13}|=\sqrt{(\frac{5}{13})^2+(\frac{1}{13})^2}=\frac{\sqrt{2}}{\sqrt{13}}$. Finally, the product is $\sqrt{13}\bullet\frac{\sqrt{2}}{\sqrt{13}}=\sqrt{2}$. Which is closest to $2\sqrt{2}$.

30.Notice that $\sqrt[3]{1000}<\sqrt[3]{2008}<\sqrt[3]{1000000}\Rightarrow 10<\sqrt[3]{2008}<100$, so the value has 2 digits to the left of the decimal.

Team Solutions

1. **A**. $R = kL \Rightarrow k = \frac{R}{L}$

   B. $x = kyz \Rightarrow (-7) = k(210)(-10) \Rightarrow k = \frac{1}{300}$

   $-2 = (\frac{1}{300})(y)(25) \Rightarrow y = -24$

   C. $V = k(\frac{T}{P}) \Rightarrow 33 = k(\frac{110}{15}) \Rightarrow k = 4.5$

   $V = 4.5(\frac{180}{28}) = \frac{405}{14}$

   D. Solving the system yields $x = \frac{5}{3}, y = \frac{1}{3}, z = \frac{-5}{3}$. Now, the variations is

   $z = k(\frac{\sqrt{x}}{y^3}) \Rightarrow \frac{-5}{3} = k(\frac{\sqrt{5/3}}{(1/3)^3}) \Rightarrow -45 = k(\frac{\sqrt{15}}{3}) \Rightarrow k = -9\sqrt{15}$

2. **A**. Completing the square yields $(x^2 - 2x + 1) + (y^2 - 2y + 1) = 2 + 1 + 1 \Rightarrow (x-1)^2 + (y-1)^2 = 4$. The shortest distance is the distance from the point to the center of the circle minus the radius so, with the point (9,5) and the center (1,1), we get $d = \sqrt{(9-1)^2 + (5-1)^2} - 2 = 4\sqrt{5} - 2$

   B. Exactly ¼ of the circle falls in the fourth quadrant, so $A = \frac{22\pi}{4} = \frac{11\pi}{2}$

   C. Completing the square yields

   $16(x^2 - 4x + 4) + 9y^2 = 80 + 64 \Rightarrow \frac{(x-2)^2}{9} + \frac{y^2}{16} = 1$. So the length of the semi-minor axis is 3 and the length of the semi-major axis is 4, so the area is $3(4)\pi = 12\pi$

   D. Completing the square yields that the equation is $\frac{(x+3)^2}{1} + \frac{(y-1)^2}{9} = 1$. Notice the center is at (-3,1). Now, the point with the highest x-value is the point directly to the right of the center. This point is (-2,1). The distance from this point to a focus is equal to the length of the half-major-axis, which is 3.

3. **A**. We can have that $2x - 3 = -7x + 5 \Rightarrow 9x = 8 \Rightarrow x = \frac{8}{9}$. Or

   $-2x + 3 = -7x + 5 \Rightarrow 5x = 2 \Rightarrow x = \frac{2}{5}$. So $\frac{8}{9} + \frac{2}{5} = \frac{58}{45}$

B. First, $x^2+7x+14=1 \Rightarrow x^2+7x+13=0$, so the product of the roots is $\frac{13}{1}=13$. Also, $x^2+7x+14=-1 \Rightarrow x^2+7x+15=0$, so the product of the roots is 15. So the total product of all of the roots is 13(15)=195.

C. Solving the first inequality we get $x \geq 2, x \leq \frac{-16}{3}$. Solving the second inequality we get -4<x<-2. Since these 2 do not intersect anywhere, there are 0 integral solutions.

D. Since none of the functions pass the vertical line test, 0 of the functions are one-to-one.

4. **A**. Slope $=\frac{7-3}{-2-3}=\frac{-4}{5}$. So the equation of the line is $(y-3)=\frac{-4}{5}(x-3) \Rightarrow 4x+5y=27$

B. The slope of the line through the given points is $\frac{-1-(-2)}{-2-5}=\frac{-1}{7}$. So the perpendicular slope is 7. The line through the given points is $(y+1)=\frac{-1}{7}(x+2)$, substituting in y=0, gives the x intercept of (-9,0). So the perpendicular line is $(y-0)=7(x+9) \Rightarrow 7x-y=-63$

C. $\frac{2(0)+4(2)+(-3)}{\sqrt{2^2+4^2}}=\frac{5}{\sqrt{20}}=\frac{\sqrt{5}}{2}$

D. Since the lines are parallel first notice that a point on the first listed line is (0,-5). Now, use the same formula as above with the point and the second line.
$\frac{2(0)+(-1)(-5)+(-3)}{\sqrt{2^2+1^2}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$

5. **A**. If the function has exactly one real root, then the discriminant is 0. So $m^2-4(-2)(-7)=0 \Rightarrow m^2=56 \Rightarrow m=\pm\sqrt{56}$. The product of the possible values for m is -56.

B. Since this is an upward opening parabola, the minimum value occurs at the vertex. The x-value of the vertex is $\frac{-(-5)}{2(3)}=\frac{5}{6}$. So the minimum value is $3(\frac{5}{6})^2-5(\frac{5}{6})+6=\frac{47}{12}$

C. Notice that the sum of the roots is $-2=\frac{-b}{a} \Rightarrow b=2a$ and the product of the roots is $-24=\frac{c}{a} \Rightarrow c=-24a$. Now the discriminant is 4, so

$b^2-4ac=4 \Rightarrow (2a)^2-4a(-24a)=4 \Rightarrow 100a^2=4 \Rightarrow a=\dfrac{1}{5}$ since a>0. Using substitution we find that $b=2(\dfrac{1}{5})=\dfrac{2}{5}$ and $c=-24(\dfrac{1}{5})=\dfrac{-24}{5}$. So the sum is $\dfrac{-21}{5}$.

D. The rational root theorem says that only possible rational roots for a function can be found by taking the factors of the constant term and dividing by the factors of the coefficient of the quadratic term. The factors of the constant term are $\pm1,\pm2,\pm4$ and the factors of the coefficient of the quadratic term are $\pm1,\pm2,\pm3,\pm6$. By dividing and taking the unique answers, we get that there are 16 possible rational roots.

6. **A**. $(2i+1)^6=[(2i+1)^2]^3=(4i-3)(4i-3)(4i-3)=(-24i-7)(4i-3)=117i+44$. So the sum of a and b is 161.
   B. When 214567 is divided by 4 the remainder is 3, so this is equivalent to $i^3=-i$
   C. The sum of all the roots of the function is 3, so the sum of the other 3 roots is 3-(1+i)=(2-i)
   D. $\dfrac{6+i}{5-2i}\bullet\dfrac{5+2i}{5+2i}=\dfrac{28+17i}{29}=\dfrac{17}{29}i+\dfrac{28}{29}$, so the sum of a and b is $\dfrac{45}{29}$

7. **A.** $\sqrt{-4x+\sqrt{-4x+\sqrt{-4x+...}}}=x+2 \Rightarrow \sqrt{-4x+(x+2)}=x+2 \Rightarrow -3x+2=(x+2)^2$
   $x^2+7x+2=0 \Rightarrow x=\dfrac{\sqrt{41}-7}{2}$. Only this value of x gives a positive value for the square root.
   B. $\dfrac{7}{6x^2+13x+6}=\dfrac{A}{2x+3}+\dfrac{B}{3x+2} \Rightarrow \dfrac{7}{6x^2+13x+6}=\dfrac{A(3x+2)+B(2x+3)}{6x^2+13x+6}$.
   So solving we get that 3A+2B=0 and 2A+3B=7, solving this system yields $A=\dfrac{-14}{5},B=\dfrac{21}{5}$. The sum is $\dfrac{7}{5}$.
   C. $\dfrac{1}{1+\dfrac{1}{x+3}}-\dfrac{1}{1+\dfrac{1}{x}}=\dfrac{3}{28} \Rightarrow \dfrac{x+3}{x+4}-\dfrac{x}{x+1}=\dfrac{3}{28} \Rightarrow \dfrac{(x+3)(x+1)-x(x+4)}{(x+4)(x+1)}=\dfrac{3}{28}$
   $\Rightarrow 28(3)=3(x^2+5x+4) \Rightarrow 3x^2+15x-72=0 \Rightarrow x=3$ since x>0.

   D. $(\sqrt{x+5}-\sqrt{2x})=0 \Rightarrow \sqrt{x+5}=\sqrt{2x} \Rightarrow x+5=2x \Rightarrow x=5$

8. **A.** $\ln(14400)=\ln(2^6\cdot3^2\cdot5^2)=6\ln(2)+2\ln(3)+2\ln(5)=6a+2b+2c$
   B. Let $u=2^x$, then this equation becomes $u^2+u-6=0 \Rightarrow u=2,-3$. The only root that makes sense is u=2, so back substituting we get x=1.
   C. After 1 year we have 1+1(.04)=1.04. After the second year we have 1.04+1.04(.04)=1.08 to 2 decimals.

D. $A = 1e^{\frac{\ln(50)}{100}(200)} = e^{2\ln(50)} = e^{\ln(2500)} = 2500$

9. **A.** Completing the square we get that $y-7 = 2(x^2 + \frac{3}{2}x + \frac{9}{16}) \Rightarrow \frac{1}{2}(y-7) = (x - \frac{3}{4})^2$. So the length of the latus rectum is ½.
   B. Substituting the values into the function, we get (a-b+c=18), (a+b+c=6) and (4a+2b+c=15). Solving this system of equations yields a=5, b=-6 and c=7. So a+b+c=6.
   C. Completing the square we get
   $4x^2 - 9y^2 - 32x - 18y + 19 = 0 \Rightarrow 4(x^2 - 8x + 16) - 9(y^2 + 2y + 1) = -19 + 64 - 9$
   $\Rightarrow \frac{(x-4)^2}{9} - \frac{(y+1)^2}{4} = 1$. So, if c is the distance between the center and a focus, we get that $c^2 = 9 + 4 \Rightarrow c = \sqrt{13}$. So the distance between the foci is $2\sqrt{13}$
   D. The point of intersection of the asymptotes of any hyperbola is the center of the hyperbola, so
   $36x^2 - y^2 + 360x + 6y - 855 = 0 \Rightarrow 36(x^2 + 10x + 25) - (y^2 - 6y + 9) = 855 + 900 - 9$
   $\Rightarrow \frac{(x+5)^2}{1746/36} - \frac{(y-3)^2}{1746} = 1$. So the center is at (-5,3).

10. **A.** $2008 = 2^3 \cdot 251$
   B. The prime factorization of $6600 = 2^3 \cdot 3^1 \cdot 5^2 \cdot 11^1$. So the number of factors is determined by (3+1)(1+1)(2+1)(1+1)=48.
   C. $\left\lfloor \frac{173}{5} \right\rfloor + \left\lfloor \frac{173}{25} \right\rfloor + \left\lfloor \frac{173}{125} \right\rfloor + \left\lfloor \frac{173}{625} \right\rfloor + \left\lfloor \frac{173}{3125} \right\rfloor = 34 + 6 + 1 + 0 + 0 + ... = 41$
   D. Notice that 27mod12=3 (since when 27 is divided by 12 there is a remainder of 3), 13mod5=3 and 87mod9=6. So 3+3-6=0. So the smallest natural number, when divided by 6 that leaves a remainder of 0 is A=6.

11. **A.** By multiplying the second equation by 2 we get -14x+6y=-78. Subtracting this equation from the first equation we get 18x=90, which reduces to x = 5. Back substituting we get that $y = \frac{-4}{3}$
   B. Use the substitution $u = \frac{1}{x}, v = \frac{1}{y}$ to transform the system to $u - v = \frac{1}{6}$ and $3u + 2v = \frac{13}{6}$. Multiplying the first equation by 2 yields $2u - 2v = \frac{2}{6}$, adding this to the second equation gives $5u = \frac{15}{6} \Rightarrow u = \frac{1}{2}$. Back substituting we see that $v = \frac{1}{3}$. Substituting these values back in to get x and y, we get x=2 and y=3.

C. Notice that the second equation can be rewritten as $(x+y)^2 = 625 \Rightarrow x+y=25$. Also, notice that the first equation can be rewritten in the following way: $(x-y)(x+y)=25 \Rightarrow (x-y)(25)=25 \Rightarrow x-y=1 \Rightarrow x=y+1$. Substituting into the second equation we get $(x+y)^2 = 625 \Rightarrow (2y+1)^2 = 625 \Rightarrow 2y+1=25 \Rightarrow y=12$. Finally with one final substitution we get that x=13.

D. If this is graphed the region turns out to be a triangle with height at the point of intersection of the two lines and base length that is the distance between the x-intercepts. The lines intersect at (-1,4) and the two x-intercepts are (1,0) and $(\frac{-7}{3},0)$. So the area is $\frac{1}{2}(4)(1-\frac{-7}{3}) = \frac{20}{3}$.

12. **A.** $\begin{vmatrix} 1 & 4 & 0 \\ -3 & 7 & 5 \\ 2 & -4 & -3 \end{vmatrix} = 1\begin{vmatrix} 7 & 5 \\ -4 & -3 \end{vmatrix} - 4\begin{vmatrix} -3 & 5 \\ 2 & -3 \end{vmatrix} = 1[7(-3)-5(-4)] - 4[(-3)(-3)-5(2)] = 3$

B. A matrix is invertible if the determinant is equal to 0. So the determinant is 0 when $-6-12m=0 \Rightarrow m=\frac{-1}{2}$

C. Det(2D-4F)=2Det(D)-4Det(F) = 2(4) – 4(-3) = 20

D. We can determine that the original system was $12x-3y=2$ and $11x+y=-3$. Solving this system gives the solution of $(\frac{-7}{45},\frac{-58}{45})$, so the sum of the solutions is $\frac{-13}{9}$.

13. **A.** True, since every function of that form is odd and all will pass the vertical line test.

B. True

C. True, Using order of operations

D. True, for example $f(x)=x^2+1$

14. **A.** $1600_7 = 1(7^3)+6(7^2)+0(7)+0(1)=637_{10}$

B. $1600_{12} = 1(12^3)+6(12^2)+0(12)+0(1)=2592_{10}$

C. $1600_{10} = 4(7^3)+4(7^2)+4(7)+4(1)=4444_7$

D. $1600_{10} = 11(12^2)+1(12)+4(1)=B14_{12}$

15. **A.** $(2x+1)(x-\frac{1}{2})^2 = (2x+1)(x^2-x+\frac{1}{4}) = 2x^3-x^2-\frac{1}{2}x+\frac{1}{4}$

B. Using long division we find the quotient to be $6x^2-2x-18+\frac{9x+53}{x^2+3}$

C. This graph has a double root at x=2 and two imaginary roots. Thus, it touches and/or crosses the x-axis only one time
D. The domain restriction comes from the denominator. Setting the denominator equal to 0 and solving we see that x=0 and x = -4 cannot be in the domain. So the domain is $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.