January 2008 Statistics Regional

Unless specified in the question, the answer must be exact. NOTA is "None of the Above".

1. The Deerfield football stadium has 8000 seats, 4500 on the Deerfield side and the rest on the visitors' side. For the game last Friday, all seats in the stadium were sold out. 63% of the Deerfield fans bought a bottle of water and 78% of the visiting fans bought a bottle of water. Given that a fan bought a bottle of water, find the probability that the fan was sitting on the Deerfield side.

a) $\frac{26}{53}$ *b)* $\frac{27}{53}$ *c)* $\frac{51}{100}$ *d)* $\frac{1116}{1600}$ *e) NOTA*

2. A new drug was given to a volunteer group of 50 people who had high blood pressure. 35 people from the group had lower blood pressure after taking the drug. What should we conclude about the effectiveness of the drug?

a) The drug was effective in 70% of the cases.
b) Since the group knew they were being given the drug, we can conclude nothing.
c) Since the 50 people were not an simple random sample, we can conclude nothing.
d), Since there was no control group that received a placebo, we can conclude nothing.
e) NOTA

3. The following are statistics about the sets X and Y: $\overline{X} = 53$, $s_X = 5$, $\overline{Y} = 71$, $s_Y = 12$. Given that X and Y are independent, find the mean and the standard deviation of the set (X+Y). The answers are in the form (mean, standard deviation).

a) (62,13) *b)* (62,17) *c)* (124,13) *d)* (124,17) *e) NOTA*

4. Preliminary research shows that 81% of respondents agreed with the statement, "I am confident that my Statistics teacher will help me pass the AP exam." How many people must be sampled to estimate the true proportion within ±0.06 with 95% confidence? Answer the question using the appropriate chart.

a) 115 *b)* 116 *c)* 164 *d)* 165 *e) NOTA*

5. Suppose 30% of students at Smith High wear glasses. Mrs. Jones is monitoring the main hallway in between classes. Let X = the number of students that must walk by until Mrs. Jones sees a student wearing glasses. The probability that a student wearing glasses walks by before X = 6 is

a) .882351 *b)* .83193 *c)* .07203 *d)* .050421 *e) NOTA*

6. We want to draw a sample of 5 without replacement from a population of 50 students who have taken the AP Statistics exam. If the students are labeled 01, 02, 03, ..., 50 and the following line is from a random digit table

73676 47150 99400 01927 27754 42648 82425 36290

Which of the following represents the sample of 5, starting from the left?

a) 6, 47, 15, 9, 1 *b)* 50, 40, 1, 42, 24 *c)* 50, 40, 19, 27, 44 *d)* 7, 36, 47, 15, 9 *e) NOTA*

January 2008 Statistics Regional

7. The following information is a summary about a data set of bowling scores from the Okahumpka Bowling League.

$\bar{x} = 174,\ s_x = 20,\ \min x = 110,\ Q_1 = 132,\ med = 150,\ Q_3 = 183,\ \max x = 250$

What is the largest integer value for an outlier on the lower end?

a) 55 *b)* 81 *c)* 99 *d)* 110 *e) NOTA*

8. A research firm wants to find the country's reaction to a speech by a politician running for President. To insure a very narrow confidence interval, they decide to interview 4000 adults. The firm randomly selects 8 states and then selects 10 zip codes at random from each state. Finally, 50 people from each zip code area are selected at random. This method of sampling is

a) simple random sample b) systematic sample c) stratified sample
d) multi-stage sample e) NOTA

9. The coefficient of determination is equal to .64. What is the value of the correlation coefficient?

a) .4096 *b)* .64 *c)* .8 *d)* −.8 *e) NOTA*

10. The mean of Mr. Morris' Algebra exam was 78. Johnny scored an 85 on the exam, which placed him at the 75.8 percentile in the class. Given that the exam scores formed a normal distribution, find the standard deviation of the scores. Use the appropriate chart to answer the question.

a) 5.306 *b)* 7 *c)* 10 *d)* 13 *e) NOTA*

11. Given the following information about two sets X and Y:
$\bar{X} = 105,\ s_X = 12,\ \bar{Y} = 150,\ s_Y = 18,\ r = .6$, find the equation of the line of best fit in slope intercept form.

a) $y = \frac{9}{10}x + \frac{111}{2}$ *b)* $y = \frac{2}{5}x + 45$ *c)* $y = \frac{9}{10}x - 30$ *d)* $y = \frac{2}{5}x + 108$ *e) NOTA*

12. Given $P(A) = .6,\ P(B) = .4,\ P(A \mid B) = .8$, find $P(B' \mid A)$.

a) no solution *b)* 1 *c)* $\frac{8}{15}$ *d)* $\frac{7}{15}$ *e) NOTA*

13. In Ms. Lambert's class, 21 students take Math, 20 students take Science and 17 students take History. 10 take Math and Science, 7 take Science and History and 9 take Math and History. 6 students take all three classes and each student takes at least one of the three classes. Find the probability that a student takes at least two classes.

a) $\frac{4}{19}$ *b)* $\frac{7}{19}$ *c)* $\frac{4}{29}$ *d)* $\frac{7}{29}$ *e) NOTA*

January 2008 Statistics Regional

14. Which of the following is NOT a condition for a chi-square test?
a) The data must be randomly selected. b) The data must be independent.
c) All expected cell counts must be at least 1. d) No more than 20% of the cells can have an expected value less than 5. e) NOTA

15. Mr. Carter is looking for more agile athletes for his basketball team. One of his requirements is that his players know how to jump rope. The probability of a student knowing how to jump rope at his school is .35. Six students try out for the team. What is the probability that at least four of the students know how to jump rope?
a) $\frac{751513}{6400000}$ *b)* $\frac{1174239}{10000000}$ *c)* $\frac{285719}{12800000}$ *d)* $\frac{1217307}{12800000}$ *e) NOTA*

16. An auto mechanic team claims that at a pit stop they can change all four tires and fill the gas tank in 15 seconds. A driver is skeptical of this claim and thinks the team can't be that fast. What are the null and alternative hypotheses that the driver should test?
a) $H_0: \mu = 15, H_a: \mu < 15$ *b)* $H_0: \mu < 15, H_a: \mu > 15$ *c)* $H_0: \mu = 15, H_a: \mu \neq 15$ *d)* $H_0: \mu = 15, H_a: \mu > 15$ *e) NOTA*

17. Find the mean of the following discrete distribution.

X	2	6	12	18	21
P(X)	.3	.1	.15	.2	.25

a) 11 *b)* 11.8 *c)* 11.85 *d)* 12 *e) NOTA*

18. Find the standard deviation of the following discrete distribution. Round your answer to two decimal places.

X	3	7	9	12	15
P(X)	.2	.16	.2	.12	.32

a) 4.60 *b)* 4.46 *c)* 4.12 *d)* 2.23 *e) NOTA*

19. David has two jars of marbles. Jar A has 5 red marbles and 7 blue marbles. Jar B has 4 red marbles and 9 blue marbles. David randomly chooses two marbles out of Jar A, one at a time and without replacement. He places both of these marbles into Jar B and then randomly chooses a marble from Jar B. Find the probability that the marble David chooses from Jar B is red.
a) $\frac{463}{1980}$ *b)* $\frac{248}{495}$ *c)* $\frac{29}{90}$ *d)* $\frac{61}{90}$ *e) NOTA*

20. Ms. Lynch gives a Pre Calculus test. The results of the test are a mean of 63 and a standard deviation of 4. She feels the results are too low, so she multiplies everyone's score by 1.5 and then subtracts 9.5 from everyone's score. Find the new mean and standard deviation of the test results. The answers are in the form (mean, standard deviation).
a) (94.5, 6) *b)* (85, 3.5) *c)* (85, 6) *d) no solution* *e) NOTA*

January 2008 Statistics Regional

21. Ujas is taking a statistics class at a local community college. The class grade consists of four categories: projects, quizzes, midterm exam and the final exam. The projects are worth 20% of the grade, quizzes are worth 30% of the grade, the midterm exam is worth 15% and the final exam is worth 35%. Ujas earns a grade of 83 on the projects, 91 on the quizzes and 80 on the midterm exam. He is taking the final exam today. What is the minimum grade Ujas needs to get on the final exam to get an average of 90 for the course?

a) 97 *b)* $\frac{682}{7}$ *c)* 98 *d)* 99 *e) NOTA*

22. When events A and B are independent, which of the following are true?

I. $P(A)P(B) = P(A \cap B)$ II. $P(A|B) = P(B)$ III. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

a) I only *b) III only* *c) II and III* *d) I, II, III* *e) NOTA*

23. 34% of students in Mu Alpha Theta are sophomores in high school. 200 members of Mu Alpha Theta are randomly selected to participate in a survey. Find the standard deviation for this situation. Round the standard deviation to three decimal places.

a) 44.880 *b)* 11.489 *c)* 8.246 *d)* 6.699 *e) NOTA*

24. Which of the following are true statements?

I. The alternative hypothesis is stated in terms of a population parameter.

II. If a sample is large enough, the necessity for it to be a simple random sample is diminished.

III. If the P-value is .05, the probability that the null hypothesis is correct is .05.

a) I only *b) II only* *c) I and II* *d) I, II, III* *e) NOTA*

25. Hayley is bored in Mr. Sleet's Statistics class. She gets out a die and starts rolling. She rolls the die 102 times before Mr. Sleet tells her to stop. The results of her rolls are as follows:

Value	1	2	3	4	5	6
Number of times	13	20	15	12	16	26

Determine the chi-square value if Hayley were to perform a goodness of fit test to determine if the die is fair.

a) 7 *b)* $\frac{7497}{1040}$ *c)* 8 *d)* 17 *e) NOTA*

26. The Spanish club at Shermer High School is 37% female. The sponsor of the club randomly chooses 150 members. Find the standard deviation of this situation. Round your solution to three decimal places.

a) 2.145 *b)* 4.602 *c)* 5.913 *d)* 34.965 *e) NOTA*

January 2008 Statistics Regional

The following information is to be used for questions 27 and 28:

The Phillip Shopkin Algebra test is given to all incoming 9th graders to determine their Algebra skills. The possible score on the test ranges from 0 to 50. A random sample of students are selected and they are given the test. The results of the test are as follows:

Gender	Number of students	Mean	Standard deviation
Male	41	28.32	4.91
Female	37	27.13	3.72

27. Using the information above, determine the t-score for the data to help determine if there is a difference between the scores of boys and girls. Assume the difference to be positive. Round the t-score to three decimal places.

a) .225 *b*) .981 *c*) 1.190 *d*) 1.213 *e*) *NOTA*

28. Using the information above, determine the degrees of freedom to be used to help determine if there is a difference between the scores of boys and girls. Use the appropriate chart to answer the question.

a) 36 *b*) 40 *c*) 74 *d*) 77 *e*) *NOTA*

29. A card is drawn from a standard deck (no jokers). Find the probability that the card is a jack or black.

a) $\frac{15}{26}$ *b*) $\frac{6}{13}$ *c*) $\frac{7}{13}$ *d*) $\frac{11}{26}$ *e*) *NOTA*

30. Find the standard deviation of the following set: {2, 6, 12, 17, 23}.

a) 7.510 *b*) 8.396 *c*) $\frac{\sqrt{1410}}{5}$ *d*) $\frac{\sqrt{282}}{2}$ *e*) *NOTA*

January 2008 Statistics Solutions

1. **B.** Of the 4500 Deerfield fans, 63%, or 2835 fans buy water. 78% of visiting fans, or 2730 fans, buy water. A total of 5565 fans bought water. Of those, 2835 were from Deerfield. So the solution is $\frac{2835}{5565} = \frac{27}{53}$.

2. **D.** Without a control group, the effectiveness of the drug can't be determined. There could be confounding variables which also might have lowered the blood pressures.

3. **C.** To find the mean of the set, you add the two means together to get the solution. To find the standard deviation of the set, square the two individual standard deviations, add them up and take the square root to get the solution.

4. **D.** To find the solution, use the formula for margin of error, which is $m = z\sqrt{\frac{p(1-p)}{n}}$. When you plug the given information in and 1.96 for z using the z-score chart, you get $.06 = 1.96\sqrt{\frac{.81(.19)}{n}}$. Solving for n gives you 164.2264, which rounds up to the answer.

5. **B.** This is a geometric situation. You must find the probabilities of seeing the first student with glasses for each of the situations from 1 student to 5 total students and add the five probabilities up to get the answer.

6. **C.** Working from the left, the first number is 50 because the other numbers before it are greater than 50. 27 appears twice, so you must skip it the second time and then 44 becomes the fifth number selected.

7. **A.** To find possible outliers, find the IQR, or $Q_3 - Q_1$. In this problem, the IQR is 51. Multiply that by 1.5 and get 76.5. Subtract that from Q_1 and get 55.5. So the greatest integer that can be an outlier is 55.

8. **D.** A multi-stage sample takes an SRS of each full subgroup of a population and then takes an SRS of the remaining subgroups.

9. **E.** The coefficient of determination is equal to r^2. Because we aren't told anything about the data set, we don't know if the correlation coefficient, r, is positive or negative.

10. **C.** To find the solution, use the z-score formula and plug in the given information along with the z-score of .7 which you get from the z score chart. You get $\frac{85-78}{sd} = .7$ which leads to the solution.

11. **A.** The formula for the line of best fit is $y - \bar{y} = r\frac{s_y}{s_x}(x - \bar{x})$. Plugging the values in gives $y - 150 = .6\left(\frac{18}{12}\right)(x - 105) \Rightarrow y - 150 = .9(x - 105) \Rightarrow y - 150 = .9x - 94.5$ which leads to the final solution.

12. **D.** First you must find the probability of the intersection using $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. Plugging the values in gives $.8 = \frac{P(A \cap B)}{.4}$. So the intersection equals .32. Therefore, A only is .28, B only is .08, and neither is .32. So,

$P(B' \mid A) = \dfrac{P(B' \cap A)}{P(A)} = \dfrac{.28}{.6} = \dfrac{7}{15}$.

13. **B.** Plug the numbers into a Venn diagram. They are too big, so subtract the number of students who take all three classes from the numbers involving two classes. You then find that 8 students do Math only, 9 do Science only and 7 do History only. The total number of students in the class is 38. There are 14 students who take two or more classes, so the solution is $\dfrac{14}{38} = \dfrac{7}{19}$.

14. **E.** All of the statements were true of chi-square testing.

15. **A.** This is a binomial situation. You must find the probability of finding 4 students, 5 students and 6 students who can jump rope individually. Then add the three probabilities together to get the exact solution.

16. **D.** Since the driver thinks it can't be that fast, he believes it is slower than 15 seconds. Therefore, the alternative is greater than 15.

17. **C.** Multiply the value by its probability and add the results together to get the solution.

18. **B.** Find the mean, which is 9.76. Subtract the mean from each value, square the differences, multiply the squared differences by their probabilities, add them up and take the square root to get the solution, which rounds to the given answer.

19. **C.** There are four possible outcomes out of Jar A. They are RR, BR, RB, and BB. Then find the probability of getting a red marble from Jar B after each of those situations. Find the probability of each situation and add the four probabilities together to get the solution.

20. **C.** When you multiply by 1.5, the mean becomes 94.5 and the standard deviation becomes 6. When you subtract 9.5 points, the mean drops to 85 and the standard deviation remains 6.

21. **B.** To find the average, first find Ujas' grade up to this point. 83(.2)+91(.3)+80(.15)=55.9. To get a 90 for the course, Ujas must earn at least 34.1 points on the final exam, which is equal to .35x. So, .35x = 34.1. The value of x is the answer.

22. **E.** The answer is I and III. I is the definition of independent events. III is the definition of union. For statement II. $P(A \mid B) = P(A)$ when A and B are independent.

23. **D.** The formula for standard deviation of a binomial distribution is $\sqrt{np(1-p)}$. Plugging the given values produces $\sqrt{200(.34)(.66)} = \sqrt{44.88} = 6.699$ to 3 decimal places

24. **A.** Hypothesis tests assume simple random samples. The p value does not give the probability that the null hypothesis is true; it gives the probability of such an extreme value, assuming the null is true.

25. **C.** The formula for chi-square is $\sum \dfrac{(obs - \exp)^2}{\exp}$. The expected value in this problem is 17, or the mean of 102 rolls. When you use the formula, you get the following: $\dfrac{(13-17)^2}{17} + \dfrac{(20-17)^2}{17} + \dfrac{(15-17)^2}{17} + \dfrac{(12-17)^2}{17} + \dfrac{(16-17)^2}{17} + \dfrac{(26-17)^2}{17}$, which equals the answer when you add the six results.

26. **C**. The formula for the standard deviation of a binomial distribution is $\sqrt{np(1-p)}$. When you plug p into the formula, you get the solution to three decimal places.

27. **D**. Use the formula for t-score which is $\frac{x - x_1}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_{x_1}^2}{n_{x_1}}}}$. When you plug the numbers in assuming the difference to be positive, you get the solution to three decimal places.

28. **A**. The degrees of freedom used is the smaller of the two sample sizes minus one when using the chart. In this case, 37-1 = 36.

29. **C**. The formula is $P(J \cup B) = P(J) + P(B) - P(J \cap B) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$.

30. **D**. Find the mean, subtract the mean from each value, square the differences, add the results up, divide by (n-1), or 4 in this case, and take the square root to get the solution.

January 2008 Team Questions

1. Use the following chart to answer each part of the question. Assume each part is independent of every other part. Assume each student takes only one of the classes.

	Pre Calculus (P)	English (E)	Chemistry (C)
Male (M)	27	30	21
Female (F)	15	30	16

Find the probability of the following:

A) $P(C \mid M)$ B) $P(E \cap F)$ C) $P(M \mid P')$

Find the exact value of the following: A + B + C

2. Roque is a tennis player who gets in 72% of his first serves. Use this information to answer the following parts. Assume each part is independent of every other part.

A) During a three set match, Roque hits 82 first serves. Find the probability that Roque gets exactly 60 first serves in. Round your answer to four decimal places.

B) The next day, Roque practices his first serve. He takes 150 first serves. Find the standard deviation of this situation. Round your answer to four decimal places.

C) During his next match, Roque hits 54 first serves. Find the probability that Roque gets at least 40 first serves in. Round your answer to four decimal places.

Find the exact value of the following: A + B + C

3. The election for the Mu Alpha Theta President is about to take place. One candidate from each of the three regions of Mu Alpha Theta is running for President. Anna is from Region 1, Bill is from Region 2, and Connie is from Region 3. Region 1 comprises 24% of the total votes for President. In Region 1, 40% of voters vote for Anna, 32% for Bill and 28% for Connie. Region 2 comprises 36% of the total votes for President. In Region 2, 34% vote for Anna, 50% vote for Bill and 16% vote for Connie. Region 3 comprises 40% of the total votes for President. In Region 3, 20% vote for Anna, 25% vote for Bill and 55% vote for Connie. Use the information to answer the following parts. Each part is independent of every other part.

A) What percent of the total vote will Anna get?

B) What percent of the total vote will Bill get?

C) What percent of the total vote will Connie get?

Find the exact value of the following: 2A + B – C

4. The height of female students at Great Falls High School is measured in inches, and the data forms a normal distribution. The mean of the data is 64 inches with a standard deviation of 4 inches. Use the information to answer the following parts. Each part is independent of every other part. Use the appropriate chart to answer each part.

A) Find the probability that a female student is taller than 70 inches.

B) Find the probability that a female student is shorter than 59 inches.

C) Find the probability that a female student has a height between 59 and 70 inches.

D) Find the probability that a female student is taller than 71 inches or shorter than 54 inches.

Find the exact value of the following: A – B + C – D

5. Given the following information about two sets X and Y:

$$\overline{X} = -12,\ s_X = 4,\ \overline{Y} = 48,\ s_Y = 9,\ r = .30$$

find the following:

A) the slope of the line of best fit

B) the y intercept of the line of best fit
C) the x intercept of the line of best fit
D) the value of the residual when X = 6 and Y = 80.
Find the exact value of the following: A + B + C + D

6. Let X = {the positive integral factors of 60}. Find the following:
A) the mean of set X. B) the median of set X. C) the interquartile range of set X.
D) the variance of set X.
Find the exact value of the following: $\frac{AD}{BC}$

7. The following is the distribution of scores from Mr. Spencer's IB Math Exam.

Score	3	4	5	6
Frequency	4	2	15	10

Find the following:
A = the mean of the distribution B = the standard deviation of the distribution
Find the exact value of the following: AB

8. Alex Jacob loves to play cards. He has a standard deck (no jokers). He draws a card from the deck, observes the card, and replaces the card within the deck. Use this information to answer the following parts. Each part is independent of every other part.
A = the probability that it takes Alex three draws before he gets an ace.
B = the probability that it takes Alex more than four draws to get a face card.
C = the probability that Alex draws a black card in three draws or less.
Find the exact value of the following: $\frac{AC}{B}$

9. Mr. Jones gives his Algebra 1 midterm. The results of the midterm are a mean of 71 with a standard deviation of 6. Mr. Jones curves the exam so that the new mean and standard deviation of the midterm is 78 and 4, respectively. This curve can be represented by the equation $y = mx + b$, where x is the original score and y is the curved score. Use the information to find the following parts. Each part is independent of every other part.
A = the value of m in the equation. B = the value of b in the equation.
C = John's curved score, given that he got a 60 on the midterm.
D = Amy's original score, given that she got an 85 after the curve.
Find the exact value of the following: A + B + C + D

10. Suppose that 73% of an simple random sample of 100 students taking a Math class in the morning like Math, while only 62% of an simple random sample of 150 students taking a Math class in the afternoon like Math. Establish a 90% confidence interval estimate for the difference between the two times. Use the appropriate chart to answer the question. Assume a positive difference and round each end of the interval to six decimal places.

11. The heights of students in Mrs. Martinez's class are normally distributed. 5.82% of the students are greater than 75 inches and 3.59% of students are less than 60 inches. Using the appropriate chart, find the following:
A = the mean of the distribution B = the standard deviation of the distribution
Find the exact value of the following: A + B

12. Given $P(A) = \frac{3}{10}$, $P(B) = \frac{3}{5}$, *and* $P(A \mid B) = \frac{1}{3}$, find the following parts. Assume each part is independent of every other part.

a = $P(B \mid A)$ b = $P(A \cup B)$ c = $P(A' \mid B')$ d = $P(B' \mid A)$

Find the exact value of the following: abcd

13. The population of Matthews High School is comprised of 40% freshman, 30% sophomores, 15% juniors and 15% seniors. Dr. Spelling's statistics class is comprised of 20 freshman, 15 sophomores, 10 juniors and 5 seniors. Find the following:

A = the chi-square value to help determine if Dr. Spelling's class is representative of the population of the school.

B = the number of degrees of freedom if a chi-square test was to be performed.

Find the exact value of the following: $\frac{A}{B}$

14. The breakdown for Mr. Hale's Math Studies class is as follows:

Grade	1	2	3	4	5	6
Percentage	.01	.01	.02	.15	.48	.33

Find the following about Mr. Hale's Math Studies class:

A = the mean of the class B = the variance of the class, rounded to two decimal places.

Find the exact value of the following: AB

15. Given the following set of data X = {10, 20. 30, 40, 50, 60, 70, 80, 90, 100}, find the following:

A = the mean of the data B = the median of the data

C = the interquartile range of the data D = the standard deviation of the data

Find the exact value of the following: $A - B + \frac{C}{D}$

January 2008 Team Solutions

1. $\frac{354355}{350558}$. For Part A, first find the number of males, and of the males, 21 are in Chemistry. So Part A = $\frac{21}{78} = \frac{7}{26}$. For Part B, there are 30 female English students and 139 total students. So Part B = $\frac{30}{139}$. For Part C, there are 97 students who are not in Pre Calculus. Of those students, 51 are male. So Part C = $\frac{51}{97}$. The sum of the three parts is equal to the solution.

2. **6.0295**. For part A, it is a binomial situation. The answer is binompdf(82, .72, 60) = .0963 to 4 decimal places. For part B, use the formula for standard deviation of a binomial situation, which is $\sqrt{np(1-p)}$. When you plug in the values, you get $\sqrt{30.24} = 5.4991$ to 4 decimal places. For part C, it is a binomial cumulative situation. The solution is 1- binomcdf (54, .72, 39) = .4341 to 4 decimal places. The sum of the three parts is the solution.

3. **60.88**. To find the total vote from each candidate, multiply the percent of the total vote for the region times the percent of the vote the candidate will get from that region. For part A, Anna will get (.24)(.4)+(.36)(.34)+.4(.2) = 29.84 percent. Bill will get .24(.32)+.36(.5)+.4(.25) = 35.68 percent. Connie will get .24(.28)+.36(.16)+.4(.55) = 34.48 percent. The solution is 2(29.84) + 35.68 – 34.48 = 60.88.

4. **.7425**. Each part is solved by using the z-score formula, where $z = \frac{raw - mean}{sd}$. For part A, the z score is 1.5, and since it's greater than, the probability is .0668. For part B, the z score is -1.25, and the probability is .1056. For part C, use the results from the first two parts to get (.9332-.1056) = .8276. For part D, there are two separate calculations and the probabilities are added together. The two z scores are 1.75 and -2.5. Using the appropriate probabilities, you get .0401 and .0062 respectively. The sum of them produces the answer for part D, or .0463. The final result from all four parts is (.0668 - .1056 + .8276 - .0463) = .7425.

5. $\frac{-467}{72}$. The formula for the line of best fit is $y - \bar{y} = r\left(\frac{s_y}{s_x}\right)(x - \bar{x})$. Plugging the values in leads to $y - 48 = .3\left(\frac{9}{4}\right)(x + 12) \rightarrow y - 48 = .675(x + 12) \rightarrow y - 48 = .675x + 8.1$. Therefore, y = .675x +56.1. So part A = .675 and part B = 56.1. Part C is solved by setting y=0. When y=0, the solution is $\frac{-748}{9}$. Part D is solved by plugging 6 in for x and the result for y is 60.15. The residual is (observed – predicted) = 80 – 60.15 = 19.85. The final solution is (.675 + 56.1 - $\frac{748}{9}$ + 19.85) = $\frac{-467}{72}$.

6. $\frac{777}{22}$. The factors of 60 are {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}. The mean of the set is 14, the median of the set is 8, the interquartile range of the set is 14 (17.5-3.5), and

the variance of the set is $\frac{3108}{11}$. To find the variance, subtract the mean from each value, square the differences, add them up, and divide by (n-1) or 11. The final result is $\frac{14(\frac{3108}{11})}{8(14)} = \frac{3108}{88} = \frac{777}{22}$.

7. $\frac{\sqrt{210}}{3}$. The mean of the distribution is 5. When you subtract the mean from each value, square the differences and add them up, you get a total of 28. Divide by (n-1), or 30, in this case, gives a variance of $\frac{14}{15}$. The standard deviation is the square root of that, or $\frac{\sqrt{210}}{15}$. When you multiply the mean by the standard deviation, you get the solution.

8. $\frac{819}{5000}$ or **.1638**. For Part A, the first two draws are not an ace and the third is. This gives you $\left(\frac{12}{13}\right)^2\left(\frac{1}{13}\right) = \frac{144}{2197}$. For part B, the formula is $(1-p)^n$. Plugging the numbers in gives you $\left(1-\frac{12}{52}\right)^4 = \frac{10000}{28561}$. For part C, the probability of drawing a black card on the first draw is .5, the second draw is .25, and the third draw is .125, for a total of .875, or $\frac{7}{8}$. Plugging the results into the expression gives you $\frac{\frac{144}{2197}\left(\frac{7}{8}\right)}{\frac{10000}{28561}} = \frac{819}{5000}$.

9. $\frac{367}{2}$ or **183.5**. To find part A, you must transform the standard deviation from 6 to 4. To do this, you must multiply by $\frac{2}{3}$. When you multiply the original mean by $\frac{2}{3}$, you get $\frac{142}{3}$. To get a new mean of 78, you must add $\frac{92}{3}$. That is part B. So the equation is in the form $y = \frac{2}{3}x + \frac{92}{3}$. To find John's curved score, plug 60 in for x and you get $\frac{212}{3}$. To find Amy's original score, plug 85 in for y and solve for x and you get 81.5. The final solution is $\frac{2}{3} + \frac{92}{3} + \frac{212}{3} + 81.5 = 183.5$.

10. **(.012103, .207897)**. The formula is $(p-p_1) \pm z\sqrt{\frac{p(1-p)}{n} + \frac{p_1(1-p_1)}{n_1}}$. Plugging the information in gives $(.73-.62) \pm 1.645\sqrt{\frac{.73(.27)}{100} + \frac{.62(.38)}{150}} = .11 \pm .0978970814$. When you find the endpoints of the interval and round appropriately, you get the solution.

11. $\mathbf{\frac{24420}{337}}$. Each piece of information is plugged into a z score formula. They are $-1.8 = \frac{60 - mean}{sd}$ *and* $1.57 = \frac{75 - mean}{sd}$ by using the z scores from the chart. Solving by elimination gives a standard deviation of $\frac{1500}{337}$. Plugging this into one of the equations produces a mean of $\frac{22920}{337}$. The sum of the two fractions is the answer.

12. $\mathbf{\frac{7}{60}}$. To solve each part, use the P(A|B) information to find that $P(A \cap B) = \frac{1}{5}$. So, the probability of A only is .1, the probability of B only is .4, and the probability of neither is .3. Use this information to find each part. Part A equals $\frac{2}{3}$, part B equals $\frac{7}{10}$, part C equals $\frac{3}{4}$ and part D equals $\frac{1}{3}$. When you multiply the four parts, you get the solution.

13. $\mathbf{\frac{5}{9}}$. The formula for chi-square is $\sum \frac{(obs - \exp)^2}{\exp}$. The observed values are given in the problem. The expected values are 20 freshman, 15 sophomores, 7.5 juniors and 7.5 seniors. Using the formula, the exact chi-square for this problem is $\frac{(20-20)^2}{20} + \frac{(15-15)^2}{15} + \frac{(10-7.5)^2}{7.5} + \frac{(5-7.5)^2}{7.5} = \frac{5}{3}$. The number of degrees of freedom is (n-1), or 3, in this problem. Therefore, when you divide the two results, you get the solution.

14. **4.1067** or $\mathbf{\frac{41067}{10000}}$. To find the mean, multiply the grade by its percentage and add the products. Your result is 5.07. To find the variance, subtract the mean from each value, square the differences, and multiply those by their respective percentages and add those products. The exact variance is .8051. However, the directions said to round to two decimal places, so part B is .81. The answer is the product of the two parts.

15. $\mathbf{\frac{\sqrt{330}}{11}}$. The mean of the data and the median of the data are 55. The interquartile range is (80-30), or 50. The standard deviation is found by subtracting the mean from each value, square the differences, add them up, divide by (n-1), or 9 in this case, and take the square root. The standard deviation is equal to $\frac{5\sqrt{330}}{3}$. When you plug the numbers into the expression and simplify completely, you get the solution.

Janaury 2008 Statistics Solutions

1. B
2. D
3. C
4. D
5. B
6. C
7. A
8. D
9. E C D
10. C
11. A
12. D
13. B
14. E
15. A
16. D
17. C
18. B
19. C
20. C
21. B
22. E
23. D
24. A
25. C
26. C
27. D
28. A E
29. C
30. D

1. $\frac{354355}{350558}$
2. 6.0295
3. 60.88
4. .7425
5. $\frac{-467}{72}$
6. $\frac{777}{22}$
7. $\frac{\sqrt{210}}{3}$
8. $\frac{819}{5000}$ or .1638
9. $\frac{367}{2}$ or 183.5
10. (.012103, .207897)
11. $\frac{24420}{337}$
12. $\frac{7}{60}$
13. $\frac{5}{9}$
14. 4.1067 or $\frac{41067}{10000}$
15. $\frac{\sqrt{330}}{11}$

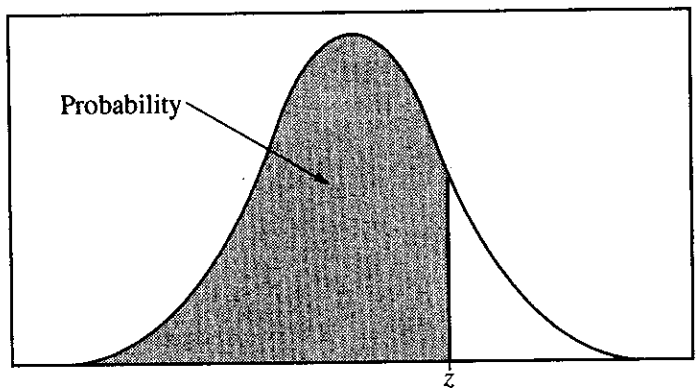

Table entry for z is the probability lying below z.

Table A *(Continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

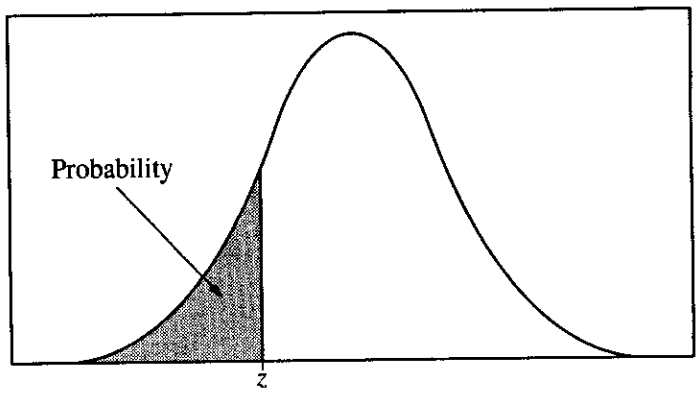

Table entry for z is the probability lying below z.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

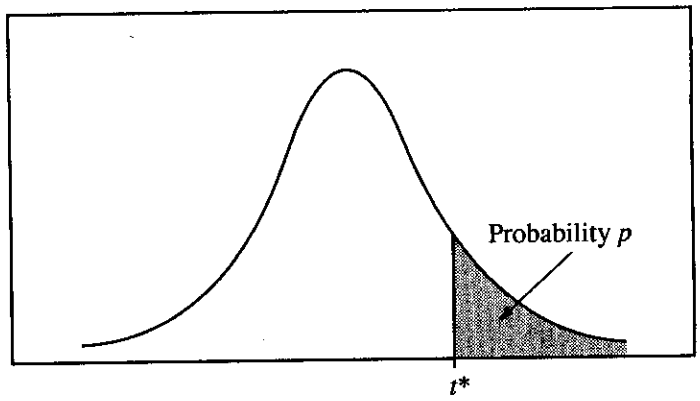

Table B t **distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

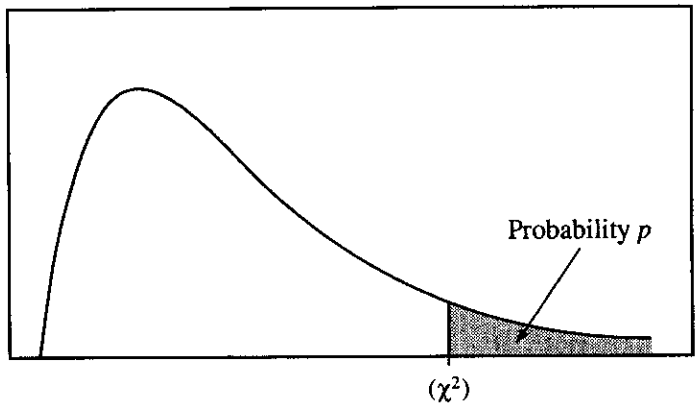

Table C χ^2 **critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2