January Regional

Geometry Individual Test

The abbreviation NOTA denotes "None of These Answers."

Diagrams may not be drawn to scale

1. The measures of two sides of a triangle are 12 inches and 13 inches. Find the range, in inches, of possible measures for the third side.

a. $1 \leq x \leq 25$ b. $1 < x < 25$
c. $x < 25$ d. $x > 1$ e. NOTA

2. Clyde wishes to center a butcher-block table in a kitchen's work triangle, that is, at a location equidistant from the refrigerator, stove, and sink. Which point of concurrency does Clyde need to locate?

a. orthocenter b. incenter
c. circumcenter d. centroid e. NOTA

3. The line through A (4, 2) and B (-1, y) is perpendicular to a line with slope -5. Find y.

a. 3 b. 1 c. 27 d. -1 e. NOTA

4. A rectangle has perimeter of 90 feet and one side with length 20 feet. What is the length, in feet, of the diagonal?

a. $10\sqrt{29}$ b. $5\sqrt{41}$
c. 45 d. 50 e. NOTA

5. If the diagonals of a rhombus are x and y units long, find the area, in square units, of the rhombus in terms of x and y.

a. $2xy$ b. $\frac{x^2+y^2}{2}$
c. $\frac{x+y}{2}$ d. $\frac{xy}{2}$ e. NOTA

6. A rectangular box measures 4 by 5 by 2 meters. What is the length, in meters, of the longest broomstick that can fit into the box?

a. $\sqrt{5}$ b. $2\sqrt{5}$
c. $3\sqrt{5}$ d. $5\sqrt{2}$ e. NOTA

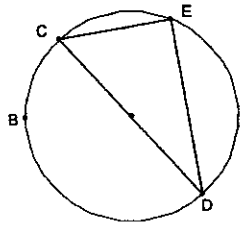
7. The measure of each interior angle of a regular polygon is eight times that of an exterior angle of the polygon. How many sides does the polygon have?

a. 16 b. 17 c. 18 d. 19 e. NOTA

8. The chairs for a concert are arranged so that each row has the same number of chairs. Teresa has 10 rows of seats in front of her, and 3 rows of seats behind her. There are 5 seats to her left and 7 to her right. How many chairs are set up?

a. 181 b. 182 c. 169 d. 168
e. NOTA

9. Given segment CD represents a diameter and minor arc ED measures 108°, determine the value of angle CDE.



a. 108° b. 36° c. 54° d. 90°
e. NOTA

10. Triangle ABC has vertices located at (3, 2), (-4, 5), and (8, 3) respectively. The triangle should be classified as _____ based on side lengths.

a. scalene b. isosceles
c. equilateral d. not a triangle
e. NOTA

11. A farmer has x feet of fencing and wants to enclose the largest area possible with the fencing. In which of the following shapes should he lay the fencing in order to maximize the area contained?

a. circle b. equilateral triangle
b. isosceles trapezoid d. square
e. NOTA

12. If one interior angle of an equiangular n-gon is 168°, how many sides does the n-gon have?

a. 24 b. 28 c. 30 d. 32
e. NOTA

13. How many times in a 24-hour day are the minute and the hour hands of a clock perpendicular?

a. 24 b. 36
c. 44 d. 48 e. NOTA

14. The hypotenuse of an isosceles right triangle has a length x. Find the perimeter of the triangle in terms of x.

a. 3x b. $x(1+\sqrt{2})$
c. $x\sqrt{2}$ d. $3x\sqrt{2}$ e. NOTA

15. A cruise ship leaves Tampa and travels 80 miles due west and then travels 150 miles due south. Upon completion of the voyage, how far, in miles, is the ship away from Tampa?

a. 230 b. 170
c. $\sqrt{230}$ d. $10\sqrt{17}$ e. NOTA

16. If 28 lines are drawn on a plane, what is the maximum number of points of intersection possible?

a. 28 b. 378 c. 392 d. infinite
e. NOTA

17. Given an isosceles triangle *XYZ*, if the area is 40cm^2 and the height is 8cm to the base, what is the length, in centimeters, of one of the two congruent sides?

a. $\sqrt{89}$ b. 10
c. 6 d. 13 e. NOTA

18. Given triangle *ABC* with vertices *A* (4, 5), *B* (-1, 4) and *C* (-2,-5), find the slope of the midsegment (defined as a segment connecting the midpoint of two consecutive sides) connecting segments AB and AC.

a. -9 b. $-\frac{1}{9}$
c. 9 d. $\frac{1}{9}$ e. NOTA

19. Two times the measure of the supplement of an angle is equal to seven times the measure of the complement of the same angle. What is the measure of the angle?

a. 36° b. 72°
c. 54° d. 110° e. NOTA

20. Consider the following statements:
    a. diagonals are congruent
    b. diagonals are perpendicular
    c. equiangular
    d. two distinct pairs of parallel sides

Which quadrilateral satisfies **all** of the following conditions?

a. trapezoid b. rhombus
c. rectangle d. square e. NOTA

21. How many of the following statements are true:

I. The exterior angle of a regular hexagon measures 60°
II. A regular 15-gon has interior angles that each measure 156°
III. If four interior angles in a pentagon measure 50°, 115°, 80°, and 120° respectively, the fifth angle must measure 175°

a. 0 b. 1 c. 2 d. 3 e. NOTA

22. Which one of the following does **not** represent a shortcut that proves two triangles are congruent?

a. SSS b. AAS
c. AA d. ASA e. NOTA

23. The needle of a circular scale in the produce department of a grocery store is 20 cm long. What distance does the tip of the needle travel when it rotates 900°? Leave your answer in terms of pi.

a. 20π b. 40π
c. 100π d. 900π e. NOTA

24. Determine the contrapositive of the following conditional statement: If a shape is a triangle, then it has three sides.

a. If a shape has three sides, then it is a triangle.
b. If a shape does not have three sides, then it is not a triangle.
c. If a shape is not a triangle, then it does not have three sides.
d. If a triangle exists it has three sides.

25. Five-foot-tall Diane casts an 84-inch shadow. How tall, to the nearest inch, is her friend if, at the same time of the day, his shadow is 1.5 feet longer than hers?

a. 6 feet, 1 inch b. 6 feet, 2 inches
c. 6 feet, 6 inches d. 5 feet, 10inches
e. NOTA

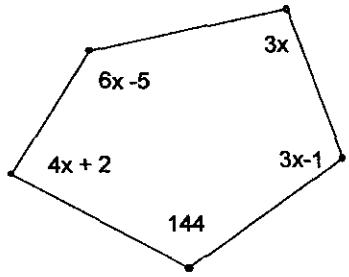
26. A convex solid has 7 faces and 21 edges, how many vertices will it have?

a. 14 b. 21
c. 16 d. 12 e. NOTA

27. The definition of a parallelogram states, "If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram." Quadrilateral *ABCD* has both pairs of opposite sides parallel. What conclusion can you make? What type of reasoning did you use?

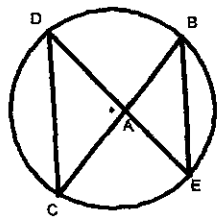
a. *ABCD* is a parallelogram; deductive
b. *ABCD* is a rectangle; inductive
c. *ABCD* is a parallelogram; inductive
d. *ABCD* is a rectangle; deductive
e. NOTA

28. In the polygon shown with degree measures given, determine the value, to the nearest tenth degree, of the largest angle:



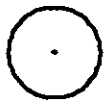
a. 165° b. 144°
c. 145° d. 115° e. NOTA

29. Find the measure of angle *CDE* if the measures of angles *CDE* and *EBC* are $(5x + 12)°$ and $(15x - 42)°$ respectively.



a. 5.4° b. 39°
c. 78° d. 27° e. NOTA

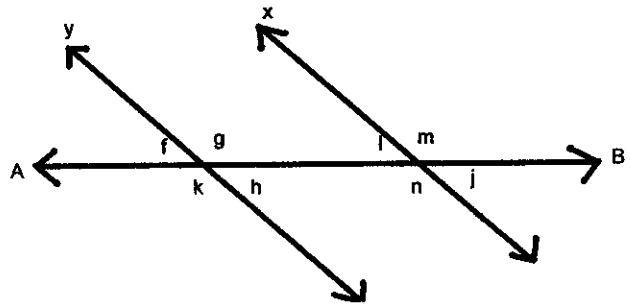
30. Identify three solid figures that could have the cross section shown below.



a. cylinder, triangular prism, cube
b. cone, rectangular prism, pyramid
c. sphere, cylinder, cone
d. cylinder, sphere, pyramid
e. NOTA

Question 1

Lines x and y are parallel. Find the number of pairs of supplementary angles in the diagram below. Then subtract from this number the amount of angles congruent to angle F.



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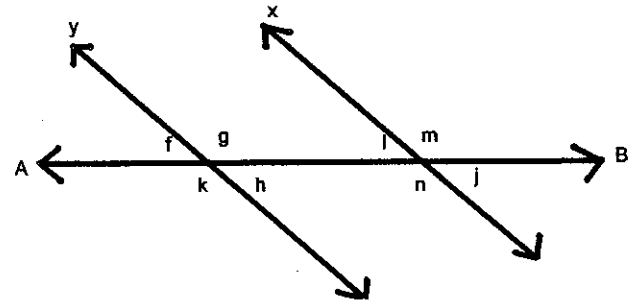

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Find the perimeter, in inches, of an equilateral triangle whose median measures 9 in (in simplest radical form).

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Question 3

Johnny has a piece of string 70 inches long. He cuts it into 3 pieces. The longest piece is twice the length of the middle-sized piece, and the shortest piece is 10 inches shorter than the middle-sized pieces. Find the ratio of the shortest to the longest piece.

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Question 4

Hector is flying a remote-controlled plane in a circle with diameter of 20 feet. The plane is traveling at a speed of 30 seconds per revolution. What is the total distance, to the nearest foot, the plane will travel in one hour? Use 3.14 for π

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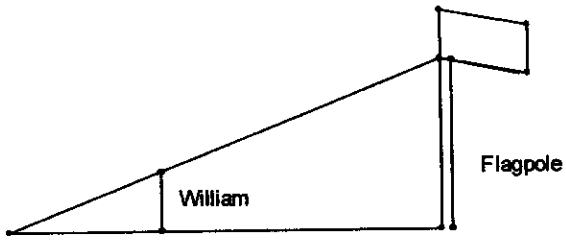
Question 5

William, who is 1.75 meters tall, wants to find the height of the school's flag pole. From the flag pole's base, he walks 15.25 meters along the pole's shadow to a position where the end of his shadow exactly overlaps the end of the flagpole's shadow. William is now 9.5 meters from the end of the shadows. How tall is the flagpole (to nearest tenth meter)?



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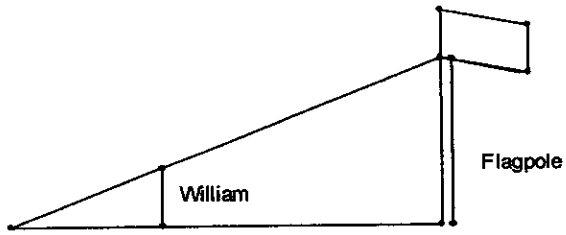
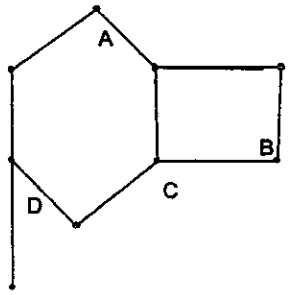


Question 6

Given the figure shown below, which is composed of a regular hexagon and square, find the value of angles $A°$, $B°$, $C°$, and $D°$. Then evaluate the following expression: $2A-BC+D^2$



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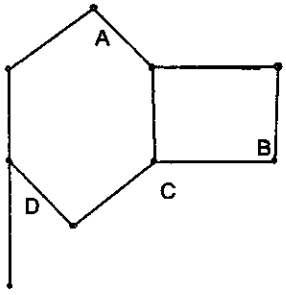

Question 7

John wants to seal his kite-shaped backyard deck. He measures the deck and finds that the diagonals measure 40 feet and 70 feet. Each gallon of sealant covers 200 ft^2, and the sealant needs to be applied every six months. Home Depot is currently advertising a special on the sealant, $19.95 per gallon. How much will John spend to protect the deck for the next three years (ignore sales tax)?

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Question 8

Let A equal the hypotenuse of a right triangle with legs that measure 8 and 15.
Let B equal the integer outside the radical when $\sqrt{80}$ is simplified.
Let C equal the area of a trapezoid with base lengths of 4 and 12 and a height measuring 10

Find 2C-(AB)

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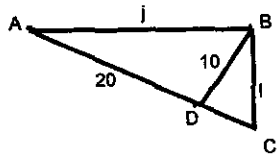

Angles *ADB*, *BDC*, and *ABC* are all right angles. Find AB + BC.

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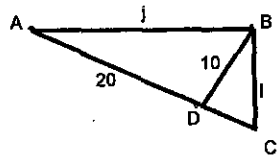

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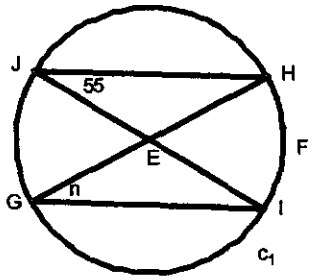

Given: In the above circle, E is the center, points J, H, I, and G are one the circle, and the radius of the circle measures 5 cm. Additionally, angle *HJI* measures 55 degrees. Find the arc length of *JH* to the nearest tenth cm. Use 3.14 for π

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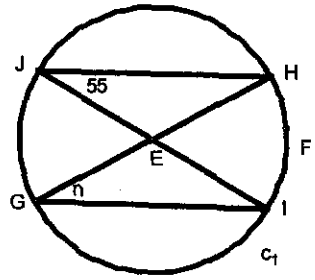
Question 10



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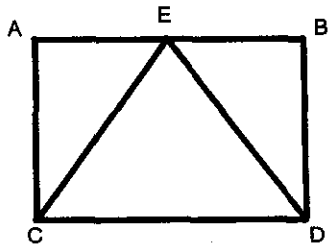

An equilateral triangle *CED* shares a side with rectangle *ACDB* as shown, with E on side AB. If side CD measures 12cm, then find the exact area of the rectangle.

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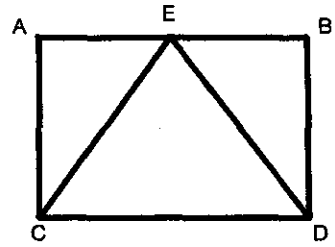

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Δ EFG has the following measures respectively: (x + 14) °, (3x) °, and (2x-5) °.
Let A equal the absolute value of the difference between the largest and smallest angle.
Let B equal the complement of angle G.
Find the value of A+B

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Question 13

Carlos builds an oblique trapezoidal prism for an engineering fair that has a base height of 4 cm and bases that measure 8 cm and 12 cm. The height of the prism is 24 cm. He finds out that in order to enter his structure in the fair, the prisms height must measure 30 cm. If all other measures on the trapezoidal base are kept constant, what is the difference in volume (cubic centimeters) between the two structures?

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Question 14

A rectangular picture is 10 in. wide by 20 in. long. Steve puts a rectangular 3-in. wide matte around the outside of the picture and then places a 4-in wide frame around the picture and matte.
Let A equal the area of the picture in square inches
Let B equal the area of the matte, without picture, in square inches
Let C equal the area of the picture frame, without picture and matte, in square inches
Find the exact value for the expression: $3(B+C) - 2A$

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Question 15

Constructing one line through a circle divides the circle into two areas. Two lines can create as many as four distinct pieces. What is the maximum number of pieces you can get by drawing four lines through a circle?

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Individual Round Solutions

1) B: The sum of the lengths of the two smaller legs of a triangle must be greater than the length of a triangle's largest side. Therefore, you perform the following two operations to find the range of values: 13-12<x and 13+12>x.

2) C: The circumcenter is the point in a triangle that is equidistant from the vertices; the location Clyde is trying to locate.

3) B: The new line would have a slope of $\frac{1}{5}$ because it is the opposite reciprocal of the original slope. Then you set up the equation, $[(y-2)/(-1-4)] = \frac{1}{5}$ and solve for y. The answer works out to be y=1.

4) B: If the perimeter is 90ft and one side equals 20ft, then its opposite side will also equal 20ft. Take the whole perimeter and subtract the measure of these two sides and you get 50ft, meaning that each of the unknown sides must measure 25ft each. To solve, you do the Pythagorean Theorem where $20^2 + 25^2 = x^2$.

5) D: The formula for the area of a rhombus is $\frac{d_1 \bullet d_2}{2}$. If the values of the diagonals are x and y respectively, when these variables are substituted, the new formula would be $\frac{xy}{2}$.

6) C: First find the diagonal of the box's base by performing the Pythagorean Theorem ($4^2 + 5^2 = d^2$), which equals $\sqrt{41}$. Next determine the value of the box's diagonal by doing: $2^2 + (\sqrt{41})^2 =$ (length of broomstick)^2. Therefore, the answer is $3\sqrt{5}$.

7) C: The equation is $\frac{(n-2)180}{n} = \frac{8(360)}{n}$. Since n appears in the denominator of both sides, this problem can be simplified to (n-2)180 = 8(360). Therefore, n=18

8) B: There are a total of 14 rows of seats, with a total of 13 seats in each row. To determine the number of seats, multiply (14)(13) =182.

9) B: Since the triangle is inscribed in a semicircle, angle E is a right angle. Since arc ED measures 108°, its corresponding inscribed angle is 54°. Therefore, the third angle must measure 180°-90°-54° = 36°.

10) A: The distances of segments AB, BC, and AC are $\sqrt{58}$, $\sqrt{148}$, and $\sqrt{26}$ respectively. Since all sides measure different lengths, the triangle is classified as scalene.

11) A: To maximize the area, the farmer should build a fence in the shape of a circle. As a polygon becomes more circular, using the formula ½ ap for area where the perimeter does not change and the length of the apothem approaches the length of the radius of the polygon its area increases; therefore, a circle would have a greater area than any of the listed polygons.

12) C: $\frac{(n-2)180}{n} = 168°$; 168n = 180n -360; -12n =-360; n= 30

13) C: During the A.M., the minute and hour hand of a clock are perpendicular a total of 22 times. Be careful because the answer is not 24 since there are two points of overlap (also called merging point) between the hours of 2:00-4:00 and 9:00-11:00. Since the clock completes the same pattern for the P.M. hours, this number is doubled. Therefore, the total number of times the hands are perpendicular is 44.

14) B: If the hypotenuse of an isosceles right triangle is x, then each leg measures $\frac{x}{\sqrt{2}}$ (or $\frac{x\sqrt{2}}{2}$). Since there are two legs, their sum is $x\sqrt{2}$. The perimeter equals $x + x\sqrt{2} = x(1+\sqrt{2})$.

15) B: The ship travels in the path of a right triangle, with the hypotenuse equaling the distance between the ship and Tampa. $80^2 + 150^2 = c^2$. Therefore, c = 170.

16) B: Use the expression $\frac{n(n-1)}{2}$ to determine the number of points of intersection. The correct answer is 378 points of intersection.

17) A: Since the height (which is also a median and altitude) is 8, the base must measure 10cm. Divide the base in half, thus each segment measures 5cm. To find the length of a congruent leg, use Pythagorean Theorem: $5^2 + 8^2 = c^2$. The answer is $\sqrt{89}$.

18) C: Find the midpoints of segments AB and AC, which are $\left(\frac{3}{2}, \frac{9}{2}\right)$ and (1, 0) respectively. The slope of the line connecting them would be found by $\frac{0-4.5}{1-1.5}$, which is simplified to 9.

19) C: This problem can be solved by creating a system of equations:

$x + y = 90°$

$x + z = 180°$

$7y = 2z$

Upon solving the system, the value of x is determined to be 54°

20) D: The definition of a square validates all of the statements (diagonals are congruent and perpendicular, equiangular, and two distinct pairs of parallel sides).

21) D: All of the statements are true.

Statement I: $\frac{360}{6} = 60°$

Statement II: $\frac{(15-2)180}{15} = 156°$

Statement III: $540° = 50° + 115° + 80° + 120° + x°$; therefore $x = 175°$

22) C: AA proves triangle similarity, but does represent a congruency shortcut.

23) C: Find the arc length $\frac{(2\pi)(20)(900°)}{360°} = 100\pi$

24) B: The contrapositive is the combination of the inverse and converse. Thus, the contrapositive is: If a shape does not have three sides, then it is not a triangle.

25) A: Use a proportion to compare the heights, $\frac{60}{84} = \frac{x}{84+18}$. The value of x is approximately 72.857, which, when rounded to the nearest inch, is 6 feet 1 inch.

26) C: Use Euler's Theorem, F+V=E+2. Therefore, the equation is 7+V=21+2. The value of V is 16.

27) A: According to the definition, the shape is a parallelogram (there is no mention of a rectangle). This form of reasoning is deductive because you use other accepted facts, rather than observe a pattern, to reach the conclusion that the shape is a parallelogram.

28) C: Determine the value of x: $6x - 5 + 3x + 3x - 1 + 144 + 4x + 2 = 540$. The value of x is 25. The largest angle measure is 145°.

29) B: Since the angles open to congruent arcs, the two angle measures must also be congruent. Therefore, $5x + 12 = 15x - 42$, meaning that $x = 5.4$. Substitute this value into one of the inscribed angle measures, which gives a degree measure of 39°.

30) C: Cylinders, spheres, and cones all contain a circular cross section.

Team Round Solutions

1) 13: There are a total of 16 pairs of supplementary angles in the diagram. There are three angles congruent to f. Therefore the difference between the values is 13.

2) $18\sqrt{3}$ in: The median of the equilateral triangle divides it into two 30°-60°-90°. If the median measures 9 in, then the short leg measures $3\sqrt{3}$ and the hypotenuse (which is also the side length of the equilateral triangle) measures $6\sqrt{3}$. The perimeter equals $3s= 3(6\sqrt{3})=18\sqrt{3}$.

3) 1:4 The shortest piece of string measures 10 inches, while the longest piece measures 40 inches. The ratio is 10:40, which reduces to 1:4.

4) 7536 feet: The circumference of the plane's flight path is 62.8ft, which it travels in 30 seconds. This number needs to be doubled to find the distance it travels in one minute (125.6 ft). This number should then be multiplied by 60 to find the total distance.

5) 4.6 m: Set up and solve the proportion $\frac{1.75}{x}=\frac{9.5}{9.5+15.25}$

6) -9660: A=120° (interior angle of regular hexagon), B=90° (interior angle of a square), C=150° (360°-120°-90°), and D=60°(exterior angle of a regular hexagon); Therefore, the equation $2(120)-(90)(150)+60^2 = -9660$.

7) \$837.90: The area of the deck is $A=\frac{40\bullet 70}{2}=1400ft^2$. It takes 7 gallons of sealant to complete the job. To protect the deck for three years, it would require 42 gallons of sealant @ \$19.95 per gallon, which equates to \$837.90.

8) 92
A = 17, Pythagorean triple
B = 4, $\sqrt{80}=4\sqrt{5}$
C = 80, Area of the trapezoid= $\frac{1}{2}(10)(4+12)=80$

9) $15\sqrt{5}$; By computing the geometric mean, the value of DC is determined to be 5. To find the value of AB and BC, use the Pythagorean Theorem.

$AB^2 = 10^2 + 20^2$, so $AB = 10\sqrt{5}$ and $BC^2 = 5^2 + 10^2$, so $BC = 5\sqrt{5}$. AB + BC = $15\sqrt{5}$.

10) 6.1cm; arc JH measures 70°, use the following formula to determine arc length $\frac{2(3.14)(5) \bullet 70°}{360°} = 6.1$.

11) $72\sqrt{3}$ cm^2: Within the triangle exists one equilateral and two 30°-60°-90° triangles. The lengths of the smaller parallel sides of the rectangle are $6\sqrt{3}$ cm. The area of the rectangle can be found by multiplying the side lengths (12 and $6\sqrt{3}$).

12) 81; x = 28.5, therefore the angles measure 42.5°, 85.5°, and 52° respectively
A= 85.5 – 42.5 = 43
B= 90°-52° = 38°
A + B = 43 + 38 =81

13) 240cm^3; The formula for the volume of a prism is BH. B=40 for both prisms so the volume of the original trapezoidal prism is 960 cm^3 (40*24), while the one with the correct height measures 1200 cm^3 (40*30). The cubic centimeter difference between the prisms is 240cm^3.

14) 1448; A= (10)(20)= 200 in^2
B= (16)(26) – 200 = 216 in^2
C= (24)(34) – 416 = 400 in^2
3(B+C) – 2A = 3(216 + 400) – 2(200) = 1448

15) 11;

Lines	1	2	3	4
Pieces	2	4	7	11