January Regional: Calculus Individual Test
Let choice E) NOTA denote "None of the Above Answers is Correct"

1) Find the range of the function $f(x)=\dfrac{1}{1+\sin^2 x}$ over the domain of all real numbers.

A) $(0,\infty)$ B) $[0, 1]$ C) $[0.5,\infty)$ D) $[0.5, 1]$ E) NOTA

2) Evaluate: $\lim_{n\to\infty}\dfrac{6^n+2^n}{8^n}$.

A) 0 B) $\dfrac{3}{4}$ C) 1 D) Diverges E) NOTA

3) A comedy lounge sells a maximum of 40 tickets per show. When the price of an individual ticket is \$20 the lounge sells all its tickets for the show. However, with each increase of \$5 in ticket price, the lounge sells one less ticket. What should the ticket price be in order to maximize profit?
A) \$90 B) \$100 C) \$110 D) \$120 E) NOTA

4) Evaluate the limit: $\lim_{x\to-\infty}\left(\dfrac{-9+x}{x}\right)^{-x/3}$

A) 0 B) e^{-3} C) e^3 D) ∞ E) NOTA

5) The function $f(x)=e^{1/(x-1)}$ has one vertical asymptote at $x=a$ and one horizontal asymptote at $y=b$. Find $a+b$.
A) -1 B) 0 C) 1 D) 2 E) NOTA

6) With an initial guess of $x_1=0$, find the second iteration of Newton's Method when calculating a root to the equation $y=x^5-5x^2+3x+3$.

A) -1 B) $-\dfrac{2}{3}$ C) $\dfrac{1}{3}$ D) 2 E) NOTA

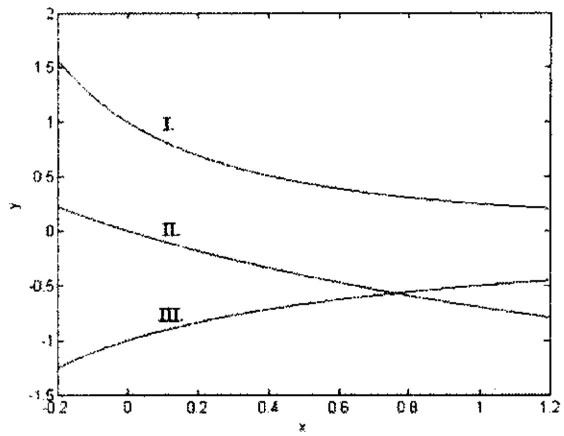
7) Find the maximum value of $\sin(\theta)+\cos\left(\theta+\dfrac{\pi}{6}\right)$ over the domain $0\le\theta\le\pi$.

A) $\dfrac{3}{4}$ B) $\dfrac{\sqrt{3}}{2}$ C) 1 D) $\dfrac{1+\sqrt{3}}{2}$ E) NOTA

8) Evaluate $\dfrac{d}{dx}\left[\dfrac{x^2+2x-1}{x^3+3x+1}\right]$ at $x=0$.

A) -5 B) -1 C) 0 D) $\dfrac{2}{3}$ E) NOTA

9) Shown are three functions I., II., and III. One of them is $f(x)$, one is $f'(x)$, and one is $f''(x)$. Which of the following correctly identifies each of the three graphs?



	$f(x)$	$f'(x)$	$f''(x)$
A)	I.	II.	III.
B)	II.	I.	III.
C)	II.	III.	I.
D)	III.	I.	II.
E)	NOTA		

10) The equation of the line tangent to $h(x) = x^3 + 2x$ when $x = 1$ is $y = Ax + B$ where A and B are real numbers. Find $A^2 + B^2$.

A) 9 B) 13 C) 18 D) 29 E) NOTA

11) Given: $\int_a^b g(x)dx = -2$, $\int_c^b g(x)dx = -3$, $\int_d^a g(x)dx = 4$

Find: $\int_c^d g(x)dx$

A) –5 B) –3 C) –1 D) 9 E) NOTA

12) Which of the following Theorems/Definitions is used to show that $\lim_{x \to \infty} \frac{\cos(x)}{x} = 0$?

A) Central Limit Theorem
B) $\delta - \varepsilon$ Limit Definition
C) Descarte's Theorem
D) Squeeze Theorem
E) NOTA

13) Find the first derivative of $g(x) = \frac{2x}{x^2 + 1}$.

A) $-\frac{2}{x^2} + 2$ B) $\frac{2}{(x^2+1)^2}$ C) $\frac{-4x^2}{(x^2+1)^2}$ D) $\frac{-4x^2+2}{(x^2+1)^2}$

E) NOTA

14) Find the x-coordinate(s) of the point(s) of inflection of the function $y = \sqrt{x^3 + 2}$.

A) $x = -2$ only
B) $x = 0$ only
C) $x = -2$ and $x = 0$
D) The function does not have any points of inflection
E) NOTA

15) Given $\int_{a}^{b}\sin(x)dx=1$. Find $\int_{a+2\pi}^{b+4\pi}\sin(x+3\pi)dx$.

A) -1 B) 0 C) 1 D) 2 E) NOTA

16) Use the linear approximation of $f(x)=\sqrt{x+1}$ near $x=0$ to approximate $\sqrt{1.02}$.

A) 1.005 B) 1.01 C) 1.015 D) 1.02 E) NOTA

17) Given $h(x)=(x+1)^2\cos^2(3x)e^{2x}(2x-1)^3$. Find $h'(0)$.

A) -2 B) 0 C) 2 D) 6 E) NOTA

18) Two lines normal to the graph $y=x^2$ are drawn. Denote line L_1 as the normal line at $x=1$ and line L_2 as the normal line at $x=2$. Find the y-coordinate of the point of intersection of lines L_1 and L_2.

A) -12 B) 2 C) $\frac{13}{2}$ D) $\frac{15}{2}$ E) NOTA

19) Given $f(x)=x^3+x+1$ and $g(x)=f^{-1}(x)$.

Evaluate: $\lim_{h\to 0}\frac{g(1+h)-g(1)}{h}$

A) $-\frac{4}{9}$ B) $\frac{1}{4}$ C) 1 D) 4 E) NOTA

20) A particle is moving along the x-axis whose position is defined by $s(t)=t^3+2t^2-4t-8$. Over which of the following time intervals is the velocity of the particle both negative and decreasing?

A) $(-\infty,2)$ B) $\left(-2,-\frac{2}{3}\right)$ C) $\left(-\frac{2}{3},\frac{2}{3}\right)$ D) $\left(-2,\frac{2}{3}\right)$ E) NOTA

21) When the volume in cubic feet of an expanding sphere is increasing four times as fast as its radius in feet, the radius is how many feet long?

A) $\frac{1}{4\pi}$ B) $\frac{1}{2\sqrt{\pi}}$ C) $\frac{1}{\pi}$ D) $\frac{1}{\sqrt{\pi}}$ E) NOTA

22) How many of the following statements are TRUE?

I. A piecewise continuous function cannot be differentiable.
II. A function that is not differentiable cannot be continuous.
III. The sum of two or more functions that are not continuous cannot be continuous.
IV. The sum of two or more functions that are not differentiable cannot be differentiable.

A) 0 B) 1 C) 2 D) 3 E) NOTA

23) Use differentials to approximate $\sqrt{7}$ given that $\sqrt{9}=3$.

A) $\frac{7}{3}$ B) $\frac{13}{5}$ C) $\frac{11}{4}$ D) $\frac{17}{6}$ E) NOTA

24) The height of a rocket launched into space is given by the function

$h(t)=\begin{cases} at^3+bt^2+ct, & \text{if } 0\le t\le 1 \\ t^2+9t-4, & \text{if } t>1 \end{cases}$ where a, b, and c are real numbers.

If the height, velocity, and acceleration are all continuous functions, find $a^2+b^2+c^2$.

A) 18 B) 98 C) 194 D) 226 E) NOTA

25) Find the second derivative of $y=x^2e^x\ln(x)$.

A) $y''=2e^x$

B) $y''=(x^2+2x)e^x\ln(x)+xe^x$

C) $y''=(2x^2+6x+2)e^x\ln(x)+(3x+3)e^x$

D) $y''=(x^2+4x+2)e^x\ln(x)+(2x+3)e^x$

E) NOTA

26) Let $f(x)$, $g(x)$, and $h(x)$ be polynomials of degree n, m, and k, such that n, m, and k are all positive integers satisfying $n>m>k$. Define the polynomial $P(x)=f(x)[g(x)+h(x)]$. What will be the degree of the polynomial of the k^{th} derivative of $P(x)$, $\frac{d^k}{dx^k}[P(x)]$?

A) $n-k$ B) $n+m-k$ C) $n+m$ D) $nm-k$ E) NOTA

27) Let $y=\sin(x^2)$ and $x=\cos(t)+\sqrt{\pi}$. Find $\frac{dy}{dt}$ when $t=\frac{\pi}{2}$.

A) $-2\sqrt{\pi}$ B) 0 C) 1 D) $2\sqrt{\pi}$ E) NOTA

28) What is the average rate of change of the function $y=x^2+2x+6$ over the domain [0, 4]?

A) 2 B) 4 C) 6 D) 8 E) NOTA

29) Given $f(x)$ is an even function and $g(x)$ is an odd function. Which of the following are even functions?

I. $\frac{d}{dx}[f(x)g(x)]$ II. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ III. $\frac{d}{dx}[f(g(x))]$ IV. $\frac{d}{dx}[g(f(x))]$

A) I. and II. only B) I. and IV. only C) II. and III. only D) I. and IV. only

E) NOTA

30) Find the n^{th} derivative of $y=xe^x$.

A) $y^{(n)}=ne^x$

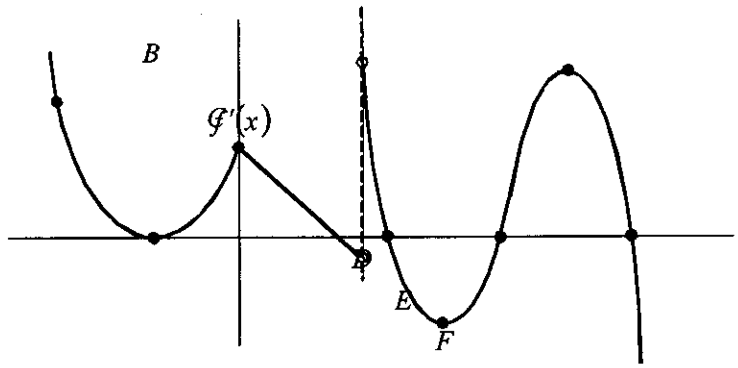
B) $y^{(n)}=(x+n)e^x$

C) $y^{(n)}=xne^x$

D) $y^{(n)}=x^ne^x$

E) NOTA

1) Shown on the graph below is the function $f'(x)$. The graph $f'(x)$ has horizontal tangents at points B, F, and H. It is also known that $f(x)$ is continuous. List the letters of the true statements in alphabetical order.



A. $f(x)$ is increasing at point A.
B. $f(x)$ is concave up at point B.
C. $f''(x)$ is differentiable at point C.
D. $f(x)$ is not differentiable at point D.
E. $f(x)$ has a relative minimum at point E.
F. $f''(x)$ is zero at point F.
G. $f(x)$ is concave down at point G.
H. $f(x)$ has an inflection point at point H.
I. $f''(x)$ is increasing at point I.

2) Given: $3x^3 + y^2 = (x-y)^2$
Find the slope of the tangent line at $x = 1$.

3) The table below gives values for $f(x)$, $g(x)$, and their derivatives $f'(x)$ and $g'(x)$.

x	1	2	3	4
$f(x)$	−3	1	4	2
$f'(x)$	−2	2	−6	−5
$g(x)$	4	3	1	−2
$g'(x)$	5	−1	−3	−1

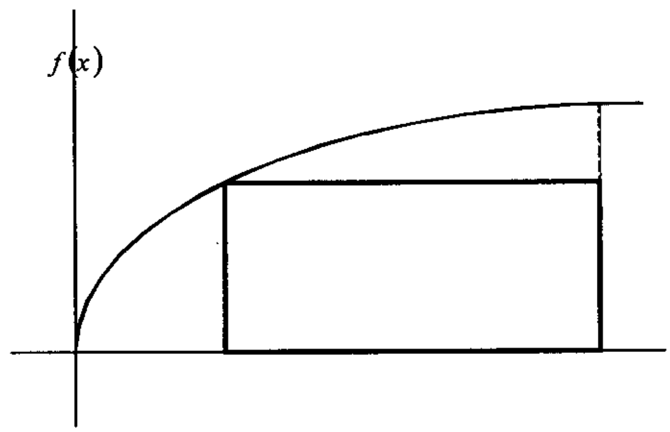
Define the following functions:

$A(x) = g\left(\frac{1}{x}\right)$ $B(x) = \frac{g(x)}{f(x)}$ $C(x) = f(g(x))$ $D(x) = g(f(x))$

Find: $A'(1) + B'(2) + C'(3) + D'(4)$

4) The base of two different solids, S_1 and S_2, is the region enclosed by the line $y = x + 1$, the x- and y-axes, and the line $x = 1$. Let V_1 be the volume of solid S_1 that has square cross sections perpendicular to the x-axis and V_2 be the volume of solid S_2 that has square cross sections perpendicular to the y-axis.
Find $V_1 - V_2$.

5) A rectangle is inscribed in the region below the graph $f(x)=\sqrt{x}$ and above the x-axis. If the rightmost edge of the rectangle lies along the vertical line $x=9$ and the bottom edge lies along the x-axis, find the maximum area of the rectangle. Write your answer in simplest radical form.



6) Given $g(x)=x^2-2x+1$ over the domain $1\le x\le 7$. Let $A=\int_1^7 g(x)dx$.

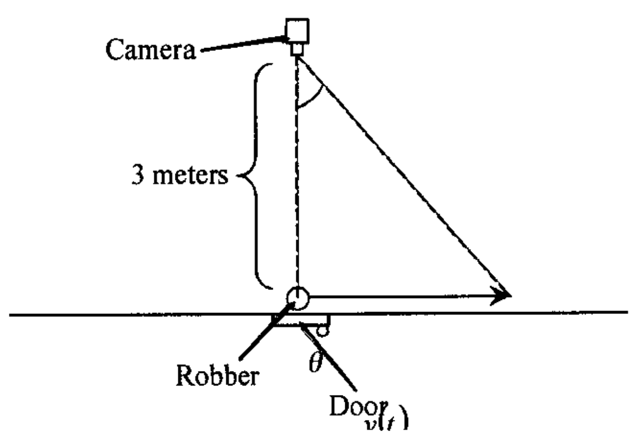
B = The approximation of A using the right endpoints of rectangles with 2 equal subdivisions.
C = The approximation of A using the left endpoints of rectangles with 6 equal subdivisions.
D = The approximation of A using the midpoints of rectangles with 3 equal subdivisions.
Find $A+B+C+D$.

7) Find the x-intercept of the line normal to the graph $h(x)=\frac{1}{3}x^2-\frac{1}{2}x+1$ that is also parallel to the line $y=-2x+9$.

8) Let $s(t)=t^2+2t-3$, $1\le t\le 5$, be the distance traveled in meters by a runner running in a straight line. An observer tracks the runner's distance starting at 1 minute and ending at 5 minutes.
A = The average velocity of the runner measured in meters/sec between $t=1$ and $t=3$.
B = The instantaneous velocity of the runner measured in meters/sec at $t=2$.
C = The time in seconds at which the instantaneous velocity of the runner measured in meters/sec equals the average velocity measured in meters/second between $t=1$ and $t=5$.
D = The distance traveled by the runner measured in meters between $t=2$ and $t=4$.
Find $A+B+C+D$.

9) A robber enters a bank by breaking into the front door. A camera is positioned such that it is initially facing the door directly 3 meters away. The robber travels with a velocity $v(t)=\frac{1}{2}t^2-\frac{1}{3}t$ where $v(t)$ is measured as the velocity traveling to the right in meters per second. The angle θ is shown.

Find the rate of change of the angle θ, $\frac{d\theta}{dt}$, in radians per second at $t=3$ seconds.



10) List the letters of the true statements in alphabetical order.

A. $\frac{d}{dx}\left[\sqrt{x-1}\right]=\frac{1}{2\sqrt{x-1}}$ for all real values of x.

B. $f(x)=x^{1/3}-1$ is differentiable everywhere.

C. The maximum value of $g(x)=-x^2+4x+4$ is 8.

D. $\lim\limits_{x\to\infty}\frac{2x-1}{\sqrt{5+2x^2}}=1$

E. A quadratic polynomial with positive real coefficients is never concave down.

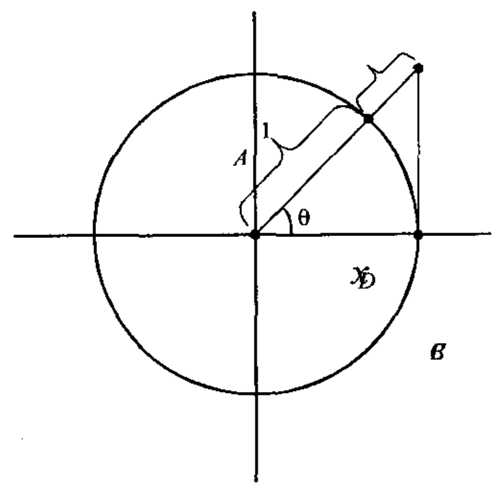
F. $\lim\limits_{x\to 2}\frac{x-2}{x^2-4}$ exists.

G. $h(x)=x^3-x$ is strictly increasing over the domain of all real numbers.

11) Given $f(x)=\sin(x)$ and $g(x)=\cos(x)$; $0<x<2008\pi$
A = The number of points of inflection of $f(x)$ over the domain $0<x<2008\pi$.
B = The number of relative minima of $f(x)$ over the domain $0<x<2008\pi$.
C = The number of relative maxima of $g(x)$ over the domain $0<x<2008\pi$.
D = The number of points where $f(x)=g(x)$ over the domain $0<x<2008\pi$.
Find: $(A+B)-(C+D)$

12) Shown is the unit circle and triangle ABC. The lengths of segments AB and AD are both equal to 1 and line segment BC is tangent to the unit circle at the point $(1,0)$. Denote the length of line segment CD as x.

Evaluate: $\lim\limits_{\theta\to 0}\frac{x}{\theta}$.



13) Find the slope of the line tangent to the graph $y=x^3-2x^2+4x-3$ at its only point of inflection.

14) Find the sum of all values of c that satisfy the Mean Value Theorem for the function $h(x)=x^3-4x^2-4x+2$ over the domain $0\le x\le 2$.

15) Given $p(x)$ is a parabola. It is known that $\int_0^1 p(x)dx=\frac{1}{3}$ and that $p(x)$ has a vertex at the point $(2,-2)$. Find $p(0)$

January Regional 2008 – Calculus Answers

Individual Test:

1)	D
2)	A
3)	C
4)	C
5)	D
6)	B
7)	C
8)	E
9)	C
10)	D
11)	A
12)	D
13)	E
14)	B
15)	A
16)	B
17)	C
18)	D
19)	C
20)	B
21)	D
22)	A
23)	E
24)	C
25)	D
26)	B
27)	D
28)	C
29)	A
30)	B

Team Test:

1)	A, D, F, H
2)	$-\frac{5}{2}$ or -2.5
3)	-1
4)	1
5)	$6\sqrt{3}$
6)	332
7)	2
8)	31
9)	$\frac{7}{12}$
10)	C, E, F
11)	0
12)	0
13)	$\frac{8}{3}$
14)	$\frac{2}{3}$
15)	2

Individual Test:

1) $\sin^2 x$ has range [0, 1]; $1+\sin^2 x$ has range [1, 2]. $\frac{1}{1+\sin^2 x}$ has range [0.5, 1] **D**

2) $\lim_{n\to\infty}\left(\frac{6}{8}\right)^n + \lim_{n\to\infty}\left(\frac{2}{8}\right)^n = 0+0=0$ **A**

3) $P(x)=(20+5x)(40-x)$

$P'(x)=(20+5x)(-1)+(5)(40-x)=0 \Rightarrow x=18$

Ticket price = $20+5(18)=$ \$110 **C**

4) $\lim_{x\to-\infty}\left(\frac{-9+x}{x}\right)^{-x/3} = \lim_{x\to-\infty}\left(1+\frac{-9}{x}\right)^{(-1/3)x}$

$\lim_{x\to-\infty}\left(1+\frac{r}{x}\right)^{nx} = e^{rn} \quad e^{(-9)(-1/3)} = e^3$ **C**

5) $\lim_{x\to1^+} e^{1/(x-1)} = +\infty$ so there is a vertical asymptote at $x=1$. $\lim_{x\to+\infty} e^{1/(x-1)} = \lim_{x\to-\infty} e^{1/(x-1)} = 1$ so there is a horizontal asymptote at $y=1$ **D**

6) $y=x^5-5x^2+3x+3,\ y'=5x^4-10x+3$

$x_2 = 0-\frac{y(0)}{y'(0)} = -\frac{3}{3} = -1$

$x_3 = -1-\frac{y(-1)}{y'(-1)} = -1-\frac{-6}{18} = -\frac{2}{3}$ **B**

7) Take derivative and set to zero.

$\cos(\theta)-\sin(\theta+\pi/6)=0$, $\cos(\theta)=\sin(\theta+\pi/6)$

$\cos(\theta)=\sin(\theta)\cos(\pi/6)+\cos(\theta)\sin(\pi/6)$

Simplifying yields $\tan(\theta)=1/\sqrt{3} \Rightarrow \theta=\pi/6$

$\sin(\pi/6)+\cos(\pi/6+\pi/6)=1$ **C**

8) $\frac{d}{dx}\left[\frac{x^2+2x-1}{x^3+3x+1}\right] =$

$\frac{(x^3+3x+1)(2x+2)-(x^2+2x-1)(3x^2+3)}{(x^3+3x+1)^2}$

Plug in $x=0$ $\quad \frac{(1)(2)-(-1)(3)}{(1)^2} = 5$ **E**

9) II. crosses from positive to negative at $x=0$, but I. and III. do not have a slope of zero at $x=0$. $\Rightarrow$ II. is $f(x)$.

II. is strictly decreasing on the interval, so $f'(x)$ must be strictly negative $\Rightarrow$ III. is $f'(x)$.

$\Rightarrow$ I. is $f''(x)$. **C**

10) $h'(x)=3x^2+2$; $\quad h(1)=3,\ h'(1)=5$

$y-3=5(x-1) \Rightarrow y=5x-2$ **D**

11) $\int_c^d g(x)dx = \int_c^b g(x)dx + \int_b^a g(x)dx + \int_a^d g(x)dx =$

$-3+2-4=-5$ **A**

12) The Central Limit Theorem and Descarte's Theorem have nothing to do with evaluating this limit. The $\delta-\varepsilon$ Limit Definition cannot be applied to a limit that approaches an infinite value. The squeeze theorem can be applied:

$-\frac{1}{x} \le \frac{\cos(x)}{x} \le \frac{1}{x}$; $\lim_{x\to\infty}\left(-\frac{1}{x}\right)=0$ and $\lim_{x\to\infty}\left(\frac{1}{x}\right)=0$

thus $\lim_{x\to\infty}\frac{\cos(x)}{x}=0$ **D**

13) $g'(x)=\frac{(x^2+1)(2)-(2x)(2x)}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2}$ **E**

14) $y' = \frac{3x^2}{2\sqrt{x^3+2}} \qquad y'' = \frac{3(x^4+8x)}{4(x^3+2)^{3/2}}$

$y''=0 \Rightarrow x^4+8x=0 \qquad x=-2$ and $x=0$

y does not exist at $x=-2$, so there is only one point of inflection at $x=0$ **B**

15) $\int_{a+2\pi}^{b+4\pi}\sin(x+3\pi)dx = \int_a^{b+2\pi}\sin(x+\pi)dx$

$=\int_a^b \sin(x+\pi)dx + \int_b^{b+2\pi}\sin(x+\pi)dx$

$=\int_a^b -\sin(x)dx + 0 = -1$ **A**

16) $f'(x)=\frac{1}{2\sqrt{x+1}}$; $f(0)=1$, $f'(0)=\frac{1}{2}$

$P_1(x)=f(0)+f'(0)(x-0)=1+x/2$

$P_1(0.02)=1+0.01=1.01$ **B**

17) $h'(x)=[2(x+1)]\cos^2(3x)e^{2x}(2x-1)^3+\ldots$

$(x+1)^2[-(2)(3)\cos(3x)\sin(3x)]e^{2x}(2x-1)^3+\ldots$

$(x+1)^2\cos^2(3x)[2e^{2x}](2x-1)^3+\ldots$

$(x+1)^2\cos^2(3x)e^{2x}[3(2)(2x-1)^2]$

$h'(0)=(2)(-1)^3+0+(2)(-1)^3+(6)(-1)^2=2$ **C**

18) L_1: $m=-1/2$, passes through $(1,1)$

$y=-x/2+3/2$

L_2: $m=-1/4$, passes through $(2,4)$

$y=-x/4+9/2$

$-x/2+3/2=-x/4+9/2 \qquad x=-12$

$y=-(-12)/2+3/2=15/2$ **D**

19) $\lim_{h\to 0}\frac{g(1+h)-g(1)}{h}=g'(1)\quad f(0)=1$

$f'(x)=3x^2+1\quad g'(1)=\frac{1}{f'(0)}=1$ **C**

20) $v(t)=3t^2+4t-4\quad a(t)=6t+4$

$v(t)$ is decreasing when $a(t)$ is negative.

$6t+4<0\Rightarrow t<-2/3$

$v(t)=(3t-2)(t+2)$. $v(t)$ is negative on the interval $\left(-2,\ \frac{2}{3}\right)$. Thus $v(t)$ is negative and decreasing over the interval $\left(-2,\ -\frac{2}{3}\right)$ **B**

21) $V=\frac{4\pi}{3}r^3\qquad \frac{dV}{dt}=4\pi\ r^2\frac{dr}{dt}$

$\frac{dV}{dt}=4\frac{dr}{dt}\Rightarrow 4\frac{dr}{dt}=4\pi\ r^2\frac{dr}{dt}\qquad r=\frac{1}{\sqrt{\pi}}$ **D**

22) I. False –

$f(x)=\begin{cases}x^2 & x<0\\ 0 & x\ge 0\end{cases},\ f'(x)=\begin{cases}2x & x<0\\ 0 & x\ge 0\end{cases}\quad f'(0)=0$

II. False – $f(x)=|x|$ which is not differentiable at $x=0$ but is continuous at $x=0$.

III. False –

$f(x)=\begin{cases}-1 & x<0\\ 1 & x\ge 0\end{cases},\ g(x)=\begin{cases}1 & x<0\\ -1 & x\ge 0\end{cases}$

$f(x)+g(x)=0$ for all x, which is continous everywhere.

IV. False – $f(x)=|x|,\ g(x)=-|x|$

$f(x)+g(x)=0$ for all x, which is differentiable everywhere. **A**

23) $f(x+\Delta x)\approx f(x)+f'(x)\Delta x;\quad f(x)=\sqrt{x}$

$\sqrt{7}\approx\sqrt{9}-\frac{2}{2\sqrt{9}}=\frac{8}{3}$ **E**

24) $h'(t)=\begin{cases}3at^2+2bt+c, & \text{if } 0\le t\le 1\\ 2t+9t, & \text{if } t>1\end{cases}$

$h''(t)=\begin{cases}6at+2b, & \text{if } 0\le t\le 1\\ 2, & \text{if } t>1\end{cases}$

$h(1),\ h'(1)$, and $h''(1)$ must all exist.

$\begin{Bmatrix}a+b+c=6\\ 3a+2b+c=11\\ 6a+b=2\end{Bmatrix}$ Solving this system gives

$(a,b,c)=(-4,13,-3)\qquad 16+169+9=194$ **C**

25) $y'=2xe^x\ln(x)+x^2e^x\ln(x)+xe^x$

$y''=\left[2e^x\ln(x)+2xe^x\ln(x)+2e^x\right]+\ldots$

$\left[2xe^x\ln(x)+x^2e^x\ln(x)+xe^x\right]+\left[e^x+xe^x\right]$

$y''=(x^2+4x+2)e^x\ln(x)+(2x+3)e^x$ **D**

26) $g(x)+h(x)$ has degree m since $m>k$. Thus $P(x)=f(x)[g(x)+h(x)]$ has degree $m+n$. By taking the k^{th} derivative of a polynomial, we subtract k from the leading coefficient to get degree $m+n-k$ **B**

27) $\frac{dy}{dt}=\frac{dy}{dx}\frac{dx}{dt}\qquad$ When $t=\pi/2,\ x=\sqrt{\pi}$

$\frac{dy}{dx}=2x\cos(x^2)\qquad \frac{dx}{dt}=-\sin(t)$

$\frac{dy}{dt}=2\sqrt{\pi}\cos(\pi)(-\sin(\pi/2))=2\sqrt{\pi}$ **D**

28) $\frac{f(4)-f(0)}{4-0}=\frac{30-6}{4}=6$ **C**

29) The derivative of an even function is odd and the derivative of an odd function is even.

I. the product of an even function and an odd function is odd. $\frac{d}{dx}[f(x)g(x)]$ is even.

II. the quotient of an even function with an odd function is odd. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ is even.

III. the composition of an even function and an odd function is even. $\frac{d}{dx}[f(g(x))]$ is odd.

IV. the composition of an odd function and an even function is even. $\frac{d}{dx}[f(g(x))]$ is odd. **A**

30) $y'=xe^x+e^x=(x+1)e^x$

$y''=(x+1)e^x+e^x=(x+2)e^x$

$\ldots\ y^{(n)}=(x+n)e^x$ **B**

Team Test:

1) A. TRUE – $f'(A) > 0$ so $f(x)$ is increasing
B. FALSE – $f''(B) = 0$ so $f(x)$ is neither concave up or concave down.
C. FALSE – $f'(C)$ is not differentiable so $f''(C)$ is not continuous nor differentiable
D. TRUE – $f'(D)$ does not exist, so $f(x)$ cannot be differentiable at point D.
E. FALSE – $f'(E) = 0$ and $f''(E) < 0$ so point E is a maximum.
F. TRUE – point F is a critical point of $f'(x)$
G. FALSE – $f''(G) > 0$ so $f(x)$ is concave up at G
H. TRUE – $f''(H) = 0$ so H is an inflection point of $f(x)$
I. FALSE – $f'(x)$ is concave down at point I so $f'''(I) < 0$, which means $f''(x)$ is decreasing at I
Answer: A, D, F, H

2) $3x^3 + y^2 = (x - y)^2 \qquad 3x^3 = x^2 - 2xy$
Plug in $x = 1$ to solve for y:
$3 = 1 - 2y \Rightarrow y = -1$
Now differentiate with respect to x:
$$9x^2 = 2x - 2y - 2x\frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{9x^2 - 2x + 2y}{-2x} = \frac{9 - 2 - 2}{-2} = -\frac{5}{2} \text{ or } -2.5$$
Answer: $-\frac{5}{2}$ or **–2.5**

3) $A'(x) = -\frac{1}{x^2}g'\left(\frac{1}{x}\right) \qquad A'(1) = -g'(1) = -5$
$$B'(x) = \frac{f(x)g'(x) - f'(x)g(x)}{f(x)^2}$$
$$B'(2) = \frac{f(2)g'(2) - f'(2)g(2)}{f(2)^2} = \frac{(1)(-1) - (2)(3)}{1^2} =$$
$= -7$
$C'(x) = f'(g(x))g'(x) \qquad C'(3) = f'(g(3))g'(3)$
$C'(3) = (-3)f'(1) = (-3)(-2) = 6$
$D'(x) = g'(f(x))f'(x) \qquad D'(4) = g'(f(4))f'(4)$
$D'(4) = (-5)g'(2) = (-5)(-1) = 5$
$-5 - 7 + 6 + 5 = -1$ **Answer: –1**

4) $V_1 = \int_0^1 (x+1)^2\,dx = \left.\frac{(x+1)^3}{3}\right|_0^1 = \frac{7}{3}$
$$V_2 = \int_0^1 1^2\,dy + \int_1^2 (2-y)^2\,dy = 1 - \left.\frac{(2-y)^3}{3}\right|_1^2 = \frac{4}{3}$$
$\frac{7}{3} - \frac{4}{3} = 1$ **Answer: 1**

5) $A(x) = \sqrt{x}(9 - x)$
$A'(x) = -\sqrt{x} + \frac{9 - x}{2\sqrt{x}} = 0 \Rightarrow x = 3 \qquad A(3) = 6\sqrt{3}$
Answer: $6\sqrt{3}$

6) $A = \int_1^7 (x-1)^2\,dx = \left.\frac{(x-1)^3}{3}\right|_1^7 = \frac{6^3}{3} = 72$
$$B = \frac{7-1}{2}[f(4) + f(7)] = 3(9 + 36) = 135$$
$$C = \frac{7-1}{6}[f(1) + f(2) + f(3) + f(4) + f(5) + f(6)]$$
$0 + 1 + 4 + 9 + 16 + 25 + 36 = 55$
$$D = \frac{7-1}{3}[f(2) + f(4) + f(6)] = 2(1 + 9 + 25) = 70$$
$72 + 135 + 55 + 70 = 332$ **Answer: 332**

7) The normal line has slope –2, so the tangent line has slope $\frac{1}{2}$ $\quad h'(x) = \frac{2}{3}x - \frac{1}{2}\cancel{x} = \frac{1}{2} \Rightarrow x = \frac{3}{2}$
$$h\left(\frac{3}{2}\right) = \frac{1}{3}\left(\frac{9}{4}\right) - \frac{1}{2}\left(\frac{3}{2}\right) + 1 = 1$$
The normal line is $y - 1 = -2\left(x - \frac{3}{2}\right)$
The x-intercept is 2. **Answer: 2**

8) $A = \frac{s(3) - s(1)}{3 - 1} = \frac{12 - 0}{2} = 6$
$B = s'(2) = 2(2) + 2 = 6$
$C =$ value of t when $s'(t) = \frac{s(5) - s(1)}{5 - 1}$
$2t = \frac{32 - 0}{4} \Rightarrow t = 3$
$D = s(4) - s(2) = 21 - 5 = 16$
$6 + 6 + 3 + 16 = 31$ **Answer: 31**

9) $\tan(\theta)=\frac{x}{3}$ where x is the distance away from the door to the right. Initially the robber is $x=0$ meters away from the door and at 3 seconds he is $\int_0^3\left(\frac{1}{2}t^2-\frac{1}{3}t\right)dt=\frac{1}{6}t^3-\frac{1}{6}t^2\Big|_0^3=3$ meters

This corresponds to $\theta=\frac{\pi}{4}$

Differentiate the first equation above with respect to time. $\sec^2(\theta)\frac{d\theta}{dt}=\frac{1}{3}v(t)$

$\frac{d\theta}{dt}=\frac{1}{3}\cos^2\left(\frac{\pi}{4}\right)v(3)=\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)=\frac{7}{12}$ rad/sec

Answer: $\frac{7}{12}$

10) A. FALSE– derivative does not exist for $x\le 1$

B. FALSE – $f'(x)=\frac{1}{3}x^{-2/3}$ does not exist at $x=0$

C. TRUE – $g'(x)=-2x+4=0\Rightarrow x=2\quad g(2)=8$

D. FALSE – $\lim\limits_{x\to\infty}\frac{2x-1}{\sqrt{5+2x^2}}=\sqrt{2}$

E. TRUE – $P(x)=ax^2+bx+c$ with $a,b,c>0$
$P''(x)=2a$ which is always greater than 0

F. TRUE – $\lim\limits_{x\to 2}\frac{x-2}{(x-2)(x+2)}=\lim\limits_{x\to 2}\frac{1}{(x+2)}=\frac{1}{4}$

G. FALSE – $h'(x)=3x^2-1<0$ when $-1/\sqrt{3}\le x\le 1/\sqrt{3}$ $h(x)$ is decreasing

Answer: C, E, F

11) A. $f''(x)=-\sin(x)$ $f''(x)=0$ when $x=n\pi$; $n=1,2,\ldots,2007$ for $0<x<2008\pi$ $\quad A=2007$

B. $f'(x)=\cos(x)$ $f'(x)=0$ when $x=\frac{(2n-1)\pi}{2}$; $n=1,2,\ldots,2008$ for $0<x<2008\pi$. There are 1004 maxima and 1004 minima. $\quad B=1004$

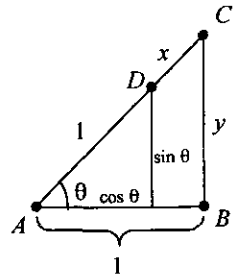
C. $g'(x)=-\sin(x)$ $g'(x)=0$ and $g(x)$ is a maximum when $x=2n\pi$; $n=1,2,\ldots,1003$ $\quad C=1003$

D. $f(x)=g(x)$ in two locations on every interval $2(n-1)\pi<x<2\pi n$; $n=1,2,\ldots,1004$ $\quad D=2008$

$(2007+1004)-(1003+2008)=0$

Answer: 0

12) Use similar triangles:



$\frac{\sin\theta}{y}=\frac{\cos\theta}{1}\quad y=\tan\theta$

$\frac{1}{\sin\theta}=\frac{x+1}{\tan\theta}\quad x=\sec\theta-1$

$\lim\limits_{\theta\to 0}\frac{\sec\theta-1}{\theta}=\lim\limits_{\theta\to 0}\frac{1-\cos\theta}{\theta\cos\theta}=$

$\lim\limits_{\theta\to 0}\left\{\left[\frac{1-\cos\theta}{\theta}\right]\left[\frac{1}{\cos\theta}\right]\right\}=(0)(1)=0$

Answer: 0

13) $y'=3x^2-4x+4\quad y''=6x-4$

$6x-4=0\Rightarrow x=2/3$

$y'=8/3$ when $x=2/3$

Answer: $\frac{8}{3}$

14) $\frac{h(2)-h(0)}{2-0}=-8$

$h'(x)=3x^2-8x-4\qquad 3c^2-8c-4=-8$

$x=2/3$ and $x=2$

The endpoint does not count, so the sum is $\frac{2}{3}$

Answer: $\frac{2}{3}$

15) $p(x)=ax^2+bx+c\qquad p'(x)=2ax+b$

$\int_0^1 p(x)dx=\frac{ax^3}{3}+\frac{bx^2}{2}+cx\Big|_0^1=\frac{a}{3}+\frac{b}{2}+c=\frac{1}{3}$

$p(2)=-2\Rightarrow 4a+2b+c=-2$

$p'(2)=0\Rightarrow 4a+b=0$

Three equations, three unknowns

$a=1,\ b=-4,\ c=2$

$p(0)=c=2$

Answer: 2