Algebra II Individual Test

January Regional

For all questions, NOTA means None Of The Aforementioned is correct.

1. Which of the following describes the graph of the following equations?

$$2x+3y=7$$
$$y=\frac{8-2x}{3}$$

a) perpendicular b) parallel c) skew d) same y-intercept e) NOTA

2. Find k such that $x^2+2(k-2)x-8k=0$ has two equal roots.

a) -2 b) 4 c) 0 or 8 d) 1 or -3 e) NOTA

3. Find the area of conic section defined by $9x^2+4y^2-36x+8y+4=0$.

a) 6π b) 9π c) 12π d) 36π e) NOTA

4. Given $(x-y)^2=121$ and $x^2+y^2=81$. Find xy.
Note: Solve over the set of real numbers.

a) -40 b) -20 c) 20 d) 40 e) NOTA

5. Evaluate: $\left|(3-4i)^{-1}\right|$, where $i=\sqrt{-1}$.

a) 1 b) 5 c) $5\sqrt{7}\ /\ 7$ d) $-5\sqrt{7}\ /\ 7$ e) NOTA

6. Identify the equation of a line passing through the point $(1,1)$ and perpendicular to the line $2x-y+3=0$.

a) $2x-y+3=0$ b) $x+2y+3=0$ c) $2x+y-3=0$

d) $x+2y-3=0$ e) NOTA

7. Find k such that $f(x)=9x^4+6x^2+8x+k$ is divisible by $(x-0)$.

a) 0 b) 1 c) 2 d) 3 e) NOTA

8. Find the range of the function $y=\left|\log_4(x+3)-2\right|+1$.

a) $y\ge 0$ b) $y\ge 1$ c) $y\ge 2$ d) $y\ge 3$ e) NOTA

9. Given: $A > 1$ and $B > 1$. $\text{Log}(A^2 + B^2) = \text{Log}(A^2) + \text{Log}(B^2)$.
Solve for B in terms of A.

a) $\left|\frac{A}{1-A^2}\right|$ b) $\frac{A}{A^2-1}$ c) $\frac{A}{\sqrt{1-A^2}}$ d) $\frac{A}{\sqrt{A^2-1}}$ e) NOTA

10. Evaluate: $(i-1)^{2008}$

a) 2^{1004} b) -2^{1004} c) 2^{2008} d) -2^{2008} e) NOTA

11. Given: $\begin{matrix} 3a-2b=4 \\ a+b-c=-1 \\ 2a-3b-3c=7 \end{matrix}$. Find $a + b + c$.

a) -4 b) -2 c) 5 d) 20 e) NOTA

12. Given $W = \frac{XY-Z}{X-1}$, $Y = ZX^{n-1}$ and $X \neq 1$. Which of the following expressions equals W?

a) $Z\left(\frac{X^n}{X-1}\right)$ b) $Z\left(\frac{X^{2n-1}-1}{X-1}\right)$ c) $Z\left(\frac{X^n-1}{X-1}\right)$ d) $Z\left(X^{n-1}-1\right)$ e) NOTA

13. Find the characteristic of $\log_{10}(12{,}181)$.

a) 10 b) 6 c) 5 d) 4 e) NOTA

14. Which of the following equations represents a line with slope a and x-intercept b.
Note: assume a and b are nonzero real numbers.

a) $(y-b) = a(x-0)$ b) $y = ax - ab$ c) $x - ay + ab = 0$

d) $y = ax + b$ e) NOTA

15. y varies directly with the square of x and inversely with the cube root of z squared.
Given $y = 5$ when $x = 2$ and $z = 8$, find the constant of proportionality.

a) 5 b) 4 c) 2 d) 1 e) NOTA

16. Let $\sqrt{9-2\sqrt{14}} = \sqrt{a} - \sqrt{b}$. Find $a + b$, where a and b are whole numbers.

a) -5 b) 5 c) 9 d) 14 e) NOTA

17. Cosmo uses Cramer's rule to solve the following system of linear equations:

$9x + 6y - 8z = 0$
$2x + 1y - 8z = 1$
$2x + 0y + 0z = 6$

Which of the following will correctly solve for z?

a) $\dfrac{\begin{vmatrix} 9 & 6 & -8 \\ 2 & 1 & -8 \\ 2 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 9 & 6 & 0 \\ 2 & 1 & 1 \\ 2 & 0 & 6 \end{vmatrix}}$ b) $\dfrac{\begin{vmatrix} 9 & 6 & 1 \\ 2 & 1 & 0 \\ 2 & 0 & 6 \end{vmatrix}}{\begin{vmatrix} 9 & 6 & -8 \\ 2 & 1 & -8 \\ 2 & 0 & 0 \end{vmatrix}}$ c) $\dfrac{\begin{vmatrix} 9 & 0 & -8 \\ 2 & 1 & -8 \\ 2 & 6 & 0 \end{vmatrix}}{\begin{vmatrix} 9 & 6 & -8 \\ 2 & 1 & -8 \\ 2 & 0 & 0 \end{vmatrix}}$ d) $\dfrac{\begin{vmatrix} 0 & 6 & -8 \\ 1 & 1 & -8 \\ 6 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 9 & 6 & -8 \\ 2 & 1 & -8 \\ 2 & 0 & 0 \end{vmatrix}}$ e) NOTA

18. The mule says to the ass, "If you give me one of your sacks, I would have as many as you would." The ass responds to the mule, "If you give me one of your sacks, I would have twice as many as you would." How many sacks do the mule and ass have total?

a) 5 b) 7 c) 12 d) 42 e) NOTA

19. Two integers are said to be "relatively prime" if they share no common positive factors other than 1. Which of the following sets are relatively prime?

I. 12, 17 II. 21, 63 III. 58, 81 IV. 17, 51 V. 13, 72

a) I, III, IV, V b) I, III, IV c) I, IV, V d) I, III, V e) NOTA

20. The graph of all functions, $y = f(x)$, are said to pass which of the following tests?

a) Horizontal Line Test b) Ratio Test c) Weierstrass' M-test

d) Vertical Line Test e) NOTA

21. Traditionally, there are ten Widgets in a Blingdoodle. If geopolitical pressure pushes the cost of one Snaggletooth to three Blingdoodles, how many Widgets will it take to purchase ten Snaggleteeth? Note: Assume geopolitical pressure only affects the cost of snaggleteeth.

a) 360 b) 300 c) 240 d) 3 e) NOTA

22. A region is bounded by the following lines: $y = x + 2$, y = 1, x = 8, $x = 1$.
Find the area of the bounded region.

a) 77 / 2 b) 91 / 2 c) 48 d) 41 e) NOTA

23. Given $f(x)=2x+1$, evaluate $\frac{1}{f^{-1}(x)}$, at $x = 2$, where $f^{-1}(x)$ is the inverse function of f.

a) 1/5 b) 1/2 c) 2 d) 5 e) NOTA

24. Solve for x over the set of real numbers: $x^2-7x+12\geq 0$.

a) $(-\infty,3]\cup[4,\infty)$ b) $(-\infty,-4]\cup[-3,\infty)$ c) $(-\infty,3)\cup(4,\infty)$

d) $(-\infty,-4)\cup(-3,\infty)$ e) NOTA

25. Given $f(a,b,c)=\frac{abc-bc^2}{a-c}$. Simplify $f(x,x,y)$, where $x\neq y$.

a) $(xy)^2/(x-y)$ b) $xy/(x-y)$ c) xy d) $xy(x-y)$ e) NOTA

26. Evaluate $\left(\frac{e^x+e^{-x}}{2}\right)^2-\left(\frac{e^x-e^{-x}}{2}\right)^2$.

a) -1 b) 0 c) 1 d) e e) NOTA

27. It takes Mary T seconds to paddle her canoe D feet upstream and back in a river where the current flows at a uniform Q feet per second. Which of the following equations can be solved for the correct speed, V, of Mary's canoe in still water?

a) $V=Q$ b) $2V=(T/D)(V^2-Q^2)$ c) $V=2D/T$

d) $V=(2D/T)(Q^2-V^2)$ e) NOTA

28. 64 unit cubes are painted blue and assembled into one large cube. The surface of the large cube is then painted orange. How many unit cubes are painted orange on at least one side?

a) 4 b) 8 c) 27 d) 38 e) NOTA

29. When J-Doke tosses a disc, its height, h, is given by the equation $h = -2t^2 + 20t + 5$, where t is measured in seconds. When J-Doke tosses his disc, how many seconds elapse before it begins to descend?

a) 1 b) 4 c) 20 d) 55 e) NOTA

30. Which of the following terms does not describe $\sqrt{2}$?

a) complex b) irrational c) real d) transcendental e) NOTA

Question #1 **Algebra II Team Round** **January Regional**

Several math students were surveyed to see which pro football team in Florida was most popular. The results showed that 5 people liked the Bucs, 3 people liked the Jaguars, 5 people liked the Dolphins, 1 person liked both the Bucs and the Jaguars, 1 person liked both the Jaguars and the Dolphins, 2 liked both the Bucs and the Dolphins, and 1 liked none of them. Nobody liked all three teams.

A: How many people were surveyed?
B: How many surveyed people do not like the Bucs?
C: How many surveyed people do not like the Dolphins?
D: How many surveyed people do not like the Jaguars?

Calculate $A + B + C + D$.

Question #2 **Algebra II Team Round** **January Regional**

Consider $f(x) = 2x^2 - Zx + 6$, where Z is an unknown integer. f is known to have a rational root at $x = a$. How many possible values are there for the value of a?

Question #3 **Algebra II Team Round** **January Regional**

Find the determinant of the following matrices:

$A = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $B = \begin{vmatrix} x & -\sqrt{1-x^2} \\ \sqrt{1-x^2} & x \end{vmatrix}$, where $-1 \le x \le 1$ $C = \begin{vmatrix} a & b \\ a & b \end{vmatrix}$ $D = \begin{vmatrix} \log 10^{-1} & 2^{\log 16} \\ \ln 1 & i^{2008} \end{vmatrix}$

Calculate $A + B + C + D$.

Question #4 **Algebra II Team Round** **January Regional**

Let the following three points define a parabolic function: (3,0); (0,12); (1,6).
A = sum of the coefficients in the form $y = ax^2 + bx + c$ (i.e. $A = a + b + c$).
B = product of the roots of the parabola.
C = sum of the roots of the parabola.
Calculate ABC.

Question #5 **Algebra II Team Round** **January Regional**

The point of intersection for each the following pairs of lines form the vertices of a polygon.

A: $x - y = 3$, $2x - y = 0$

B: $y - 10 = 3(x+1)$, $y - 10 = -\frac{1}{2}(x+1)$

C: $y = \frac{2}{3}x + 2$, $y = -5x - 15$

D: $2x + 5y + 7 = 0$, $3x - y + 2 = 0$

Find the area of the polygon $ABCD$.

Question #6 **Algebra II Team Round** **January Regional**

$F(x) = |x|$ and $G(x) = 5 - 2|x - 2|$.
The graphs of $F(x)$ and $G(x)$ intersect at two points, (a, b) and (c, d). Calculate $a + b + c + d$.

Question #7 **Algebra II Team Round** **January Regional**

Let the operation ♦ be defined as follows: $x \blacklozenge n = n(x)^{n-1}$.
Evaluate the following at $x = 2$:

$A = x \blacklozenge -1$ $B = x \blacklozenge 0$ $C = x \blacklozenge 1$ $D = x \blacklozenge 2$
Calculate $4(A + B + C + D)$.

Question #8 **Algebra II Team Round** **January Regional**
$\log_{256} x - \log_x 4 = 3/4$. Find the product of all x which are solutions to the given equation.

Question #9 **Algebra II Team Round** **January Regional**
Convert each to base 10.
A: 1234_6 B: 12_{12} C: 31_8 D: 101.01_2
Find the units digit of the product $ABCD$.

Question #10 **Algebra II Team Round** **January Regional**
Find the area enclosed by the following conic sections:
A: $x^2 + y^2 - 2x + 16y = 12$ B: $x^2 + y^2 - 4x - 8y = 0$
C: $100x^2 + 9y^2 - 200x + 90y = 575$ D: $x^2 + 4y^2 + 2x + 56y = -161$
Calculate $(A + B + C + D)/\pi$.

Question #11 **Algebra II Team Round** **January Regional**
Find the remainder for each of the following polynomial divisions:
A: $\dfrac{x^3 + x - 21}{x - 3}$ B: $\dfrac{x^2 + 5x + 10}{x + 4}$ C: $\dfrac{x^3 - 2x^2 + x + 6}{x - 2}$ D: $\dfrac{x^2 + 2x + 1}{x + 1}$
Calculate $A + B + C + D$.

Question #12 **Algebra II Team Round** **January Regional**
Boyle's law states that the pressure and volume of an ideal gas are inversely proportional. Assume air behaves as an ideal gas. If the volume of air in a balloon is 56 cubic inches when the pressure is 18 psi, what will be the new volume (in cubic inches) when the pressure is reduced to 16 psi?

Question #13 **Algebra II Team Round** **January Regional**
$A = \log_2\left(2\sqrt{50} - \sqrt{128}\right)$
B = the value of x such that $\log_2(\log_3(\log_4 x)) = 0$
C = the value of y such that $\log_a(y-1) + \log_a(y+3) = \log_{\sqrt{a}}(y+2)$, where $a > 0$.
D = the value of z such that $36^{\log_6 3} = 12z - 3$
Calculate $ABCD$.

Question #14 **Algebra II Team Round** **January Regional**
A runner wishes to strengthen a mixture that is 20% Gatorade® to one that is 80% Gatorade®. How many liters of Gatorade® should be added to 10 liters of the 20% mixture?

Question #15 **Algebra II Team Round** **January Regional**
Calculate xy, where $x^2 + y^2 = 15$ and $x - y = 3$.

Algebra II Individual Answer Key

1. B	2. A	3. A	4. B	5. E
6. D	7. A	8. B	9. D	10. A
11. B	12. C	13. D	14. B	15. A
16. C	17. E	18. C	19. D	20. D
21. B	22. A	23. C	24. A	25. C
26. C	27. B	28. E	29. E	30. D

Algebra II Team Round Answer Key

1. 27	2. 12	3. 1
4. 504	5. 17	6. 6
7. 19	8. 64	9. 5
10. 145	11. 23	12. 63
13. -336	14. 30	15. 3

1. B Lines have the same slope (-2/3), but different y-int. Thus… **Parallel**	2. A $[2(k-2)]^2-4(1)(-8k)=0$ $4(k^2-4k+4)+32k=0$ $k^2+4k+4=0$ $k=-2$	3. A Conic is an ellipse with equation $\left(\frac{x-2}{2}\right)^2+\left(\frac{y+1}{3}\right)^2=1$ Area = (2)(3)π = 6π
4. B $(x-y)^2=x^2-2xy+y^2$ $=81-2xy=121$ $xy=-20$	5. E $\left\|\frac{1}{3-4i}\right\|=\left\|\frac{1}{3-4i}\frac{3+4i}{3+4i}\right\|$ $=\left\|\frac{3+4i}{25}\right\|=\frac{\|3+4i\|}{25}$ $=\frac{\sqrt{3^2+4^2}}{25}=\frac{5}{25}$ $1/5$	6. D slope = -1/2 point = (1,1) equation = $(y-1)=-1/2(x-1)$ $x+2y-3=0$
7. A Constant term must be zero for the polynomial to be divisible by x. Thus… 0	8. B Graph has a minimum at (13,1) then $g(x)$ increases without bound as x increases. Thus the range is… $y\geq 1$	9. D $A^2B^2=A^2+B^2$ $A^2=B^2(A^2-1)$ $B^2=A^2/(A^2-1)$ $B=A/\sqrt{A^2-1}$
10. A $(i-1)^{2008}=[(i-1)^2]^{1004}$ $=[-2i]^{1004}$ $=(-2)^{1004}i^{1004}$ $=2^{1004}$	11. B Consider… $\left.\begin{array}{r}3a-2b=4\\ a+b-c=-1\\ 2a-3b-3c=7\end{array}\right\}\equiv\left\{\begin{array}{l}Eq1\\ Eq2\\ Eq3\end{array}\right.$ *Eq* 1+*Eq* 2–*Eq* 3 = 2(*a*+*b*+c)= –4→ -2	12. C $W=\frac{XY-Z}{X-1}$ $=\frac{X(ZX^{n-1})-Z}{X-1}$ $Z(X^n-1)/(X-1)$
13. D log 10^4<log 12181<log 10^5 4< log 12181 < 5 characteristic = 4	14. B slope = a, point = $(b,0)$ → $(y-0)=a(x-b)$ or… $y=ax-ab$	15. A $y=k\frac{x^2}{\sqrt[3]{z^2}}$ $5=k\frac{4}{4}$ $k=5$

16. C

$9-2\sqrt{14}=a+b-2\sqrt{ab}$

$a+b=9$ & $ab=14$

$(9-b)b=14$

$b=7, a=2$

or $b=7$, $a=2$

$\boxed{a+b=9}$

17. E

$$z=\frac{\begin{vmatrix}9&6&0\\2&1&1\\2&0&6\end{vmatrix}}{\begin{vmatrix}9&6&-8\\2&1&-8\\2&0&0\end{vmatrix}}$$

18. C

$x+1=y-1$

$2(x-1)=y+1$

$x=5$ and $y=7$

$\boxed{x+y=12}$

19. D

I. relatively prime
II. multiples of 3
III. relatively prime
IV. multiples of 17
V. relatively prime

I, III, V

20. D

All **functions** have at most one y assigned for every x. Thus, they pass the…

Vertical Line Test

21. B

$W=10S\left(\frac{3B}{S}\right)\left(\frac{10W}{B}\right)$

$=\boxed{300}$

22. A

bounded region is a trapezoid.

$A=\left(\frac{y_2+y_1}{2}\right)\Delta x$

$=\left(\frac{2+9}{2}\right)(8-1)$

$=\boxed{77/2}$

23. C

$f^1(x)=(x-1)/2$

at $x=2$, $f^1(x)=½$.

$1/f^1=\boxed{2}$

24. A

$(x-3)(x-4)\geq 0$

Quadratic is equal to zero at $x=3$ and 4; negative between the roots.

$\boxed{(-\infty,3]\cup[4,\infty)}$

25. C

$f=\frac{x^2y-xy^2}{x-y}=\frac{xy(x-y)}{x-y}$

$\boxed{xy}$

26. C

$\left(\frac{e^x+e^{-x}}{2}\right)^2-\left(\frac{e^x-e^{-x}}{2}\right)^2$

$=\frac{1}{4}\left[\begin{matrix}(e^{2x}+e^{-2x}+2)\\-(e^{2x}+e^{-2x}-2)\end{matrix}\right]$

$=\frac{1}{4}[4]$

$=\boxed{1}$

27. B

$\left(\frac{D}{V+Q}\right)+\left(\frac{D}{V-Q}\right)=T$

$\rightarrow \boxed{2V=(T/D)(V^2-Q^2)}$

28. E

Surface of cube is 4 by 4. 56 cubes are on the surface of the large cube.

$\boxed{56}$

29. E

looking for vertex of the parabola…

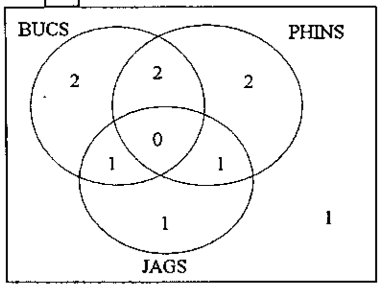
t_{max} = -b / 2a = -20 / (2)(-2)

t_{max} = 5 sec

30. D

$\sqrt{2}$ is a root of a polynomial with integer coefficients (i.e. $x^2-1=0$). Thus, it is not transcendental.

1. 27



$A = 10, B = 5, C = 5, D = 7$

2. 12

Possibilities include the following:

$\pm 6/1, \pm 6/2, \pm 3/1, \pm 3/2, \pm 2/1, \pm 2/2, \pm 1/1, \pm$

eliminating repetitions…

$\pm 6, \pm 3, \pm 2, \pm 3/2, \pm 1, \pm 1/2$

3. 1

$A = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = \underline{1}.$

$B = \begin{vmatrix} x & -\sqrt{1-x^2} \\ \sqrt{1-x^2} & x \end{vmatrix} = x^2 + (1-x^2) = \underline{1}.$

$C = \begin{vmatrix} a & b \\ a & b \end{vmatrix} = ab - ab = \underline{0}.$

$D = \begin{vmatrix} \log 10^{-1} & 2^{\log 16} \\ \ln 1 & i^{2008} \end{vmatrix} = (-1)(1) - 0 = \underline{-1}.$

4. 504

$\left.\begin{array}{l} 9a + 3b + c = 0 \\ c = 12 \\ a + b + c = 6 \end{array}\right\} \Rightarrow a = 1, b = -7$

$roots\ of\ y = x^2 - 7x + 12\ are\ x = 3, 4.$

$\underline{A = 6}, \underline{B = 12}, \underline{C = 7}.$

5. 17

$A: (-3,-6)$
$B: (-1,10)$
$C: (-3,0)$
$D: (-1,-1)$

The vertices form a trapezoid with bases = 6,11 and height = 2. → Area = $\underline{17}$.

6. 6

$|x| = 5 - 2|x-1|$

$\Rightarrow \begin{cases} x = 5 - 2(x-2) \\ -x = 5 + 2(x-2) \end{cases}$

$\Rightarrow \begin{cases} x = 3, y = 3 \\ x = -1/3, y = +1/3 \end{cases}$

$(a,b) = \underline{(3,3)}$ & $(c,d) = \underline{(-1/3,1/3)}$.

7. 19

$A = -1(2)^{-2} = \underline{-1/4}.$

$B = 0(2)^{-1} = \underline{0}.$

$C = 1(2)^0 = \underline{1}.$

$D = 2(2)^1 = \underline{4}.$

$4(A + B + C + D) = 4(11/4) = 19.$

8. 64

let $\log x = z \log 2$

$\rightarrow \rightarrow \frac{z}{8} - \frac{2}{z} = \frac{3}{4} \rightarrow \frac{z^2 - 16 = 6z}{8z}$

$\rightarrow z^2 - 6z - 16 = 0 \rightarrow z = 8, -2$

Thus $z = -2, 8$.

Product of solutions = $2^8 2^{-2} = \underline{64}$

9. **5** $A=1234_6=6^3+2(6^2)+3(6^1)+4(6^0)=310_{10}$. $B=12_{12}=12^1+2=14_{10}$. $C=31_8=3(8^1)+1=25_{10}$. $D=101.01_2=2^2+2^0+2^{-2}=5.25_{10}$. $ABCD=(31^1)(7^2)(5^3)(3^1)$ **Note:** odd multiple of 5. Units digit = 5	10. **145** $A: (x-1)^2+(y+8)^2=77$. → Area = 77π. $B: (x-2)^2+(y-4)^2=20$. → Area = 20π. $C: \left(\frac{x-1}{3}\right)^2+\left(\frac{y+5}{10}\right)^2=1$. → Area = 30π. $D: \left(\frac{x+1}{6}\right)^2+\left(\frac{y+7}{3}\right)^2=1$. → Area = 18π.
11. **23** $A=\frac{x^3+x-21}{x-3}=x^2+3x+10+\frac{9}{x-3}\to 9$. $B=\frac{x^3+x-21}{x-3}=x+1+\frac{6}{x+4}\to 6$. $C=\frac{x^3+x-21}{x-3}=x^2+1+\frac{8}{x-2}\to 8$. $D=\frac{x^3+x-21}{x-3}=x+1+\frac{0}{x+1}\to 0$.	12. **63** $V \propto 1/p \Rightarrow p_1V_1=p_2V_2$ $V_2=V_1\left(\frac{p_1}{p_2}\right)=56\left(\frac{18}{16}\right)$ $V_2=63$ in^3
13. **-336** $A=\log_2\left(10\sqrt{2}-8\sqrt{2}\right)=\log_2\left(2\sqrt{2}\right)=3/2$ $B: \log_2(\log_3(\log_4 x))=0$ → $x=4^3=64$ $C: (y-1)(y+3)=(y+2)^2$ $2y-3=4y+4$ → $y=-7/2$ $D=9=12z-3$ → $z=1$. $ABCD=-336$	14. **30** $(0.2[10]+x)/(10+x)=0.8$ $0.2[10]+x=0.8[10]+0.8x$ $0.2x=0.6[10]$ $x=30$ liters
15. **3** $xy=0.5\,([x^2+y^2]-[x-y]^2)$ $=0.5\,(15-3^2)=3$.	