For all questions, answer E. means none of the above answers is correct.

1. Let $f(x)=3^x$ what is $f'(x)$?

A) $\frac{3^x}{\ln 3}$ B) 3^x C) $3^x \ln 3$ D) $3^{x\ln 3}$ E) NOTA

2. Let $f(x)=\sqrt{x^2+2x-3}+\ln(x-1)$. What is the range of $f(x)$?

A) $(-\infty,-3]\cup[1,\infty)$ B) $[1,\infty)$ C) $(1,\infty)$
D) $(-\infty,\infty)$ E) NOTA

3. What is the slope of the tangent line to the curve $y^2=x^3$ when $y=8$?

A) -48 B) 3 C) $4\sqrt{3}$ D) 48 E) NOTA

4. What is the average value of 3^x on the interval [0,2] rounded to the nearest tenth?

A) 4.0 B) 4.1 C) 4.4 D) 8.8 E) NOTA

5. What is the volume of the solid formed by revolving the finite region bound by the curves $x=1, y=-1,\ x=3$, and $y=x$ around the line $x=-1$?

A) $\frac{8\pi}{3}$ B) $\frac{40\pi}{3}$ C) $\frac{56\pi}{3}$ D) $\frac{112\pi}{3}$ E) NOTA

6. Evaluate: $\lim_{x\to 0}\frac{x^2-2x+1}{\cos x}$

A) -1 B) 0 C) 1 D) ∞ E) NOTA

7. Given: $f(x)=3\arcsin\frac{x}{5}$. What is $f'(4)$?

A) $-1/3$ B) $\frac{3}{\sqrt{41}}$ C) $1/3$ D) 1 E) NOTA

8. What is the y-intercept of the tangent line to $y=2^x+\sin x$ at $x=3$ (rounded to the nearest tenth)?

A) -5.5 B) 11.5 C) 21.8 D) 27.7 E) NOTA

9. A farmer has a barn with a wall 60 feet long. He is going to use some or all of the wall, along with 60 feet of fencing to form a rectangular enclosure. What is the maximum area he can fence in?

A) 225 B) 400 C) 450 D) 525 E) NOTA

10. Given: $f(x) = 3^{x^2+1}$. What is $f'(-1)$?

A) $-9\ln 2$ B) 9 C) $9\ln 2$ D) $18\ln 2$ E) NOTA

11. What does the second derivative test indicate about the function $f(x) = 3 - x^4$ at $x = 0$?

A) $f(x)$ is at a local minimum
B) $f(x)$ is concave up
C) $f(x)$ has a point of inflection
D) the test is inconclusive
E) NOTA

12. Evaluate: $\int_0^{\sqrt{2}} \sqrt{4 - x^2}\, dx$.

A) 1 B) $1 + \frac{\pi}{2}$ C) $2\sqrt{2}$ D) 2π E) NOTA

13. Given $f(x) = x^2 + \cos x + \frac{1}{x+4}$ what is the average value of $f'(x)$ on the interval $[-\pi, \pi]$?

A) $\frac{2\pi}{4 - \pi^2}$ B) $\frac{1}{\pi^2 - 16}$ C) $\frac{\pi^2 + \pi - 16}{\pi(\pi^2 - 16)}$
D) $\frac{2\pi}{16 - \pi^2}$ E) NOTA

14. Evaluate the limit, rounding to the nearest hundredth: $\lim_{x \to \frac{\pi}{2}} \frac{3 - \log_{(\pi/2)} x^3}{\cos x}$

A) -4.23 B) -1.23 C) 1.23 D) 4.23 E) NOTA

15. What is the area enclosed by the cardioid described in polar coordinates as $r(\theta) = 1 + \cos\theta$?

A) π B) $\frac{3\pi}{2}$ C) $\frac{5\pi}{2}$ D) 3π E) NOTA

16. Evaluate using the trapezoidal rule with $n = 4$ (round to the nearest tenth): $\int_0^2 x^3 dx$

A) 2.3 B) 4.0 C) 4.3 D) 6.3 E) NOTA

17. The normal line to $y = 5 - x^3$ at $x = 3$ intersects the line $y = 2x - 1$ at the point (a,b). What is $a + b$ rounded to the nearest tenth?

A) -33.3 B) -22.5 C) -10.8 D) 5.6 E) NOTA

18. Let $f(x) = \sin^2(2x+5)$. What is $f'(x)$?

A) $2\sin(2x+5)$ B) $4\sin(2x+5)$ C) $2\sin(2x+5)\cos(2x+5)$
D) $4\sin(2x+5)\cos(2x+5)$ E) NOTA

19. Evaluate the indefinite integral: $\int (2x^3 + e^{2x}) dx$

A) $\frac{x^4 + e^{2x}}{2} + C$ B) $\frac{x^4}{4} + \frac{e^{2x}}{2} + C$ C) $2x^4 + e^{2x} + C$
D) $6x^2 + 2e^{2x} + C$ E) NOTA

20. Let $f(x) = \sqrt{4 - x^2}$. What is the domain of $f'(x)$?

A) $(-2,2)$ B) $[-2,2]$ C) $(-\infty,-2) \cup (2,\infty)$
D) $(-\infty,-2] \cup [2,\infty)$ E) NOTA

21. Evaluate: $\lim_{x \to \infty} \frac{8x^2 - 5x + 6}{2x^2 - 1}$

A) 0 B) 4 C) 8 D) ∞ E) NOTA

22. A cube has sides of length 2.00. Use differentials to determine the propagated error in a calculation of the cube's volume caused by a .05 deviation in the side length. Round to the nearest hundredth.

A) .59 B) .60 C) .62 D) .63 E) NOTA

23. A solid has square cross sections perpendicular to the x-axis with an edge in the region bound by $y = x^2$, the x-axis, $x = 1$ and $x = 3$. What is its volume?

A) $\frac{26}{3}$ B) 20 C) 32 D) $\frac{242}{5}$ E) NOTA

24. Let $f(x)=\dfrac{e^x}{\ln x}$. What is $f'(x)$?

A) $\dfrac{e^x(x\ln x-1)}{x(\ln x)}$ B) $\dfrac{e^x(x\ln x-1)}{(\ln x)^2}$ C) $\dfrac{e^x(x\ln x-1)}{x(\ln x)^2}$

D) $\dfrac{e^x(\ln x-1)}{x(\ln x)^2}$ E) NOTA

25. Let $f(x)=\int_a^{x^2}\sqrt{t}\cos t dt$. What is $f'\left(\sqrt{\pi}\right)$?

A) -2π B) $-\sqrt{\pi}$ C) $\sqrt{\pi}$ D) 4π E) NOTA

26. Evaluate: $\int\left(x+x^{-1}\right)dx$

A) $\dfrac{x^2}{2}+\ln|x|+C$ B) $\dfrac{x^2}{2}+\log|x|+C$ C) $x^2+\ln|x|+C$

D) $x^2+\log|x|+C$ E) NOTA

27. A particle is at rest up until time $t=0$, and subsequently its velocity is described by the equation $v(t)=t^3+\cos t$. What is the displacement of the particle from its initial position at time $t=2$ (round to the nearest tenth)?

A) 2.4 B) 3.2 C) 3.9 D) 4.6 E) NOTA

28. Let $f(x)=|x|$. Evaluate: $\lim_{x\to 0^-} f'(x)$

A) -1 B) 0 C) 1 D) does not exist E) NOTA

29. Evaluate: $\lim_{x\to 2}\dfrac{x^2+x+6}{x-2}$

A) 0 B) 2 C) 5 D) does not exist E) NOTA

30. Given: $f(x)=(2x+1)(x-3)$. What is $f'(2)$?

A) 3 B) 8 C) 13 D) 15 E) NOTA

February Calc Regional Team Questions

Q1) $f(x)=x^3+5x+1$ $g(x)=4x^4+x^2$ $A=f'(1)$ $B=f''(2)$ $C=g'(0)$ $D=g''(-1)$ What is $\int_C^D x\cos(Ax^2+B)dx$, rounded to the nearest thousandth?

Q2) $A=\int_0^{\pi}\sin^2 x dx$ $B=\int_2^3 x\sqrt{x-1}dx$ $C=\int_{-1}^{1}3^x dx$ $D=\int_1^e \ln x dx$

What is $A\cdot B\cdot C\cdot D$ rounded to the nearest tenth?

Q3) A 3-dimensional solid has cross sections perpendicular to the x-axis that are equilateral triangles with an altitude in the region bound by $y=9-x^2$ and $y=x^2-9$. What is its volume?

Q4) Let $f(x)=1.3^x$ $A=$ the area between $f(x)$ and the x-axis on [0,4]
$B=$ the area between $f(x)$ and the x-axis on [0,4], using the left endpoint approximation with 8 subintervals
$C=$ the area between $f(x)$ and the x-axis on [0,4], using the right endpoint approximation with 8 subintervals
$D=$ the area between $f(x)$ and the x-axis on [0,4], using the midpoint approximation with 8 subintervals What is $A+B+C+D$ rounded to the nearest hundredth?

Q5) $A=\int_1^{\infty}\frac{dx}{x^5}$ $B=\int_0^{A\pi}\frac{(\cos 2x+\sec^2 x)}{3}dx$ $C=\int_{\ln B}^{0}e^{-x}dx$ $D=\int_C^{e^2}\frac{\ln x}{x}dx$ What is D?

Q6) An engineer is designing a box with the following constraints: the height must be 3 feet, the surface area must be 32 square feet, and the width must be greater than or equal to the length. In cubic feet, what is the maximum achievable volume of the box?

Q7) Let $f(x)=\begin{cases}\sin\frac{x}{3} & x<0\\ x^2+5x & 0\le x<2\\ 2^{-x}(32)\ln 2 & 2<x<4\\ \frac{1}{x\ln 4} & x>4\end{cases}$ Evaluate: $\int_{-\pi}^{8}f(x)dx$

Q8) A 6 foot tall man is walking directly away from a 20 foot tall light post at 7 feet per second. At what rate is the length of his shadow increasing when he is 30 feet away from the pole in feet per second?

Q9) Given: $f(x)=x\ln x$ $g(x)=\frac{3^x}{\cos x}$. $A=f'(g(1))$ $B=f(g'(2))$ $C=f''(g(2))$ $D=g'(f'(3))$ What is $A\cdot B\cdot C\cdot D$ rounded to the nearest integer?

February Calc Regional Team Questions

Q10) A particle that is traveling along the x-axis has a velocity that is a function of time, given by the equation $v(t) = t^2 e^t$ for all real-valued times.

W = the distance travled by the particle between $t = 0$ and $t = 2$
X = the velocity of the particle at time $t = 2$ Y = the acceleration of the particle at time $t = 2$
Z = the total length of time that the particle is decelerating (i.e. $a < 0$)

What is $W - X - Y + Z$?

Q11) $f(x) = x^3 - 6x + 2$ $g(x) = 3x^4 + 4x^3 - 12x^2 - 8$
A = the maximum value of $f(x)$ on $[-2,2]$ B = the minimum value of $f(x)$ on $[-2,2]$
C = the maximum value of $g(x)$ on $[-3,2]$ D = the minimum value of $g(x)$ on $[-3,2]$

What is $\dfrac{C+D}{A+B}$?

Q12) $f(x) = \sec x + x$ $g(x) = \sqrt{x}\tan 2x$

P = the slope of the tangent line to $f(x)$ at $x = \dfrac{\pi}{6}$ Q = the slope of the normal line to $f(x)$ at $x = \pi$

R = the slope of the tangent line to $g(x)$ at $x = 4\pi$ S = the slope of the normal line to $g(x)$ at $x = \dfrac{\pi}{2}$

What is $P \cdot Q \cdot R \cdot S$?

Q13) $f(x) = \cos \pi x + 8x^3 + 2^x \ln 2$
A = the mean value of $f(x)$ on $[-1,0]$ B = the mean value of $f(x)$ on $[0,1]$
C = the mean value of $f(x)$ on $[-2,0]$ D = the mean value of $f(x)$ on $[0,2]$

What is $\dfrac{ABC}{D}$ rounded to the nearest integer?

Q14) $f(x) = \dfrac{3x^2 + 13x - 10}{x^2 + 3x - 10}$ $L = \lim\limits_{x \to \infty} f(x)$ $M = \lim\limits_{x \to 0} f(x)$ $N = \lim\limits_{x \to -5} f(x)$ $O = \lim\limits_{x \to -\infty} f(x)$

What is $\dfrac{LM}{NO}$?

Q15) $f(x) = \dfrac{3^x \tan \pi x}{x}$ $g(x) = \dfrac{\ln 2^x}{\sec \pi x}$ What is $f'(3) - g'(3)$?

Individual Test

1. C
2. D
3. B
4. E
5. D
6. C
7. D
8. A
9. C
10. E
11. D
12. B
13. B
14. D
15. B
16. C
17. A
18. D
19. A
20. A
21. B
22. B
23. D
24. C
25. A
26. A
27. E
28. A
29. D
30. A

Team Round

1. .037
2. 11.7
3. $\frac{1296\sqrt{3}}{5}$
4. 28.31
5. 2
6. 12
7. $\frac{53}{3}$
8. 3
9. -114
10. $-10e^2$
11. -4
12. $\frac{10\sqrt{2}}{3}$
13. 4
14. $\frac{7}{17}$
15. $9\pi + \ln 2$

1. $f(x)=3^x=e^{x\ln 3}$ $\quad f'(x)=\ln 3e^{x\ln 3}=3^x\ln 3$ **C**

2. Domain of $\sqrt{x^2+2x-3}=\sqrt{(x+3)(x-1)}$: $(-\infty,-3]\cup[1,\infty)$
   Domain of $\ln(x-1)$: $(1,\infty)$ Common domain: $(1,\infty)$
   Range on domain: $(-\infty,\infty)$ **D**

3. $x=y^{2/3}=8^{2/3}=4$

   $2y\dfrac{dy}{dx}=3x^2\Rightarrow\dfrac{dy}{dx}=\dfrac{3x^2}{2y}\Rightarrow\left.\dfrac{dy}{dx}\right|^{(x,y)=(4,8)}=3$ **B**

4. $\dfrac{1}{2-0}\int_0^2 3^x\,dx=\dfrac{1}{2}\dfrac{3^x}{\ln 3}\Big|_0^2=\dfrac{8}{2\ln 3}\approx 3.6$ **E**

5. $V=\int_1^3 2\pi(x+1)(x+1)dx=2\pi\int_1^3(x+1)^2\,dx=2\pi\dfrac{(x+1)^3}{3}\Big|_1^3=\dfrac{112\pi}{3}$ **D**

6. $\dfrac{x^2-2x+1}{\cos x}$ is continuous at $x=0$, so $\lim\limits_{x\to 0}\dfrac{x^2-2x+1}{\cos x}=\left.\dfrac{x^2-2x+1}{\cos x}\right|_{x=0}=1$ **C**

7. $f'(x)=\dfrac{3}{5\sqrt{1-(x/5)^2}}\Rightarrow f'(4)=1$ **D**

8. $\dfrac{dy}{dx}=2^x\ln 2+\cos x$ $\quad y\big|_{x=3}=8+\sin 3$ $\quad \left.\dfrac{dy}{dx}\right|_{x=3}=8\ln 2+\cos 3$

   $y-y_1=m(x-x_1)\Rightarrow y-(8+\sin 3)=(8\ln 2+\cos 3)(x-3)$
   $y(0)=(8+\sin 3)-3(8\ln 2+\cos 3)\approx -5.5$ **A**

9. $Perimeter=2(height)+width=2h+w=60\Rightarrow w=60-2h$

   $A=w\cdot h=(60-2h)h\Rightarrow\dfrac{dA}{dh}=60-4h$

   Extrema: $\dfrac{dA}{dh}=0=60-4h\Rightarrow h=15\Rightarrow A=450$ **C**

10. $f'(x)=(2x)(\ln 3)3^{x^2+1}$ $\quad f'(-1)=-18\ln 3$ **E**

11. $f'(x)=-4x^3$ $\quad f'(0)=0$
    $f''(x)=-12x^2$ $\quad f''(0)=0$ Test is inconclusive for $f''(0)=0$. **D**

12. Let $x = 2\sin\theta \quad \Rightarrow dx = 2\cos\theta d\theta \quad x = 0 \Rightarrow \theta = 0 \quad x = \sqrt{2} \Rightarrow \theta = \pi/4$

$$\int_0^{\sqrt{2}} \sqrt{4-x^2}\,dx = \int_0^{\pi/4} \sqrt{4-4\sin^2\theta}\,2\cos\theta d\theta = 4\int_0^{\pi/4} \sqrt{\cos^2\theta}\cos\theta d\theta = 4\int_0^{\pi/4} \cos^2\theta d\theta$$

$$\ldots = 2\int_0^{\pi/4} [1+\cos 2\theta]d\theta = 2\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/4} = \frac{\pi}{2}+1 \quad \textbf{B}$$

13. $$\frac{f(\pi)-f(-\pi)}{\pi-(-\pi)} = \frac{\frac{1}{4+\pi} - \frac{1}{4-\pi}}{2\pi} = \frac{1}{\pi^2-16} \quad \textbf{B}$$

14. $3(1-\log_{(\pi/2)}(\pi/2)) = 0 \qquad \cos(\pi/2) = 0 \qquad \Rightarrow$ L'Hospital

$$\lim_{x\to\frac{\pi}{2}} \frac{3(1-\log_{(\pi/2)} x)}{\cos x} = 3\lim_{x\to\frac{\pi}{2}} \frac{-1/x\ln(\pi/2)}{-\sin x} = \frac{3}{\sin(\pi/2)\cdot\pi/2\cdot\ln(\pi/2)} \approx 4.23 \quad \textbf{D}$$

15. $$Area = \frac{1}{2}\int_\alpha^\beta [r(\theta)]^2 d\theta = \frac{1}{2}\int_0^{2\pi} [1+\cos\theta]^2 d\theta = \frac{1}{2}\int_0^{2\pi} [1+2\cos\theta+\cos^2\theta]d\theta =$$

$$\ldots = \frac{1}{2}\int_0^{2\pi}\left[1+2\cos\theta+\frac{1+\cos 2\theta}{2}\right]d\theta = \frac{1}{2}\left[\theta+2\sin\theta+\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right]_0^{2\pi} = \frac{3\pi}{2} \quad \textbf{B}$$

16. $$I \approx \frac{1}{2}\frac{2}{4}[f(0)+2f(.5)+2f(1)+2f(1.5)+f(2)]$$

$$= \frac{1}{4}[0+2(.125)+2(1)+2(3.375)+8] \approx 4.3 \quad \textbf{C}$$

17. $(a,b) \Rightarrow b = 2a-1 \Rightarrow a+b = 3a-1$

$y = 5-x^3 \qquad y(3) = -22$

$\frac{dy}{dx} = -3x^2 \qquad \left.\frac{dy}{dx}\right|_{x=3} = -27 \qquad m_\perp = -\frac{1}{dy/dx} = \frac{1}{27}$

$y+22 = \frac{1}{27}(x-3) \qquad y = 2x-1 \qquad \Rightarrow 2a-1 = \frac{1}{27}(a-3)-22 \Rightarrow a \approx -10.7457$

$a+b = 3a-1 = 3(-10.7457\ldots)-1 \approx -33.3 \quad \textbf{A}$

18. $f'(x) = 2\sin(2x+5)\cos(2x+5)(2) = 4\sin(2x+5)\cos(2x+5)$ **D**

19. $$\int (2x^3+e^{2x})dx = 2\frac{x^4}{4}+\frac{e^{2x}}{2}+C = \frac{x^4+e^{2x}}{2}+C \quad \textbf{A}$$

20. $f'(x)=\dfrac{-2x}{2\sqrt{4-x^2}}$ $4-x^2>0 \Rightarrow -2<x<2$ **A**

21. Only highest order terms play a role approaching infinity: $\lim\limits_{x\to\infty}\dfrac{8x^2}{2x^2}=4$ **B**

22. $V=s^3$ $\dfrac{dV}{ds}=3s^2$ $dV=3s^2\,ds$ $dV=3(2)^2(.05)=.60$ **B**

23. $V=\int_1^3 (x^2)^2\,dx=\int_1^3 x^4\,dx=\left.\dfrac{x^5}{5}\right|_1^3=\dfrac{242}{5}$ **D**

24. $f'(x)=\dfrac{\ln x\cdot e^x-e^x\cdot 1/x}{(\ln x)^2}=\dfrac{e^x(x\ln x-1)}{x(\ln x)^2}$ **C**

25. $\dfrac{d}{dx}\int_a^{x^2}\sqrt{t}\cos t\,dt=2x\left(\sqrt{x^2}\cos x^2\right)=2x^2\cos x^2 \Rightarrow \left[2x^2\cos x^2\right]_{x=\sqrt{\pi}}=-2\pi$ **A**

26. $\int\left(x+x^{-1}\right)dx=\dfrac{x^2}{2}+\ln|x|+C$ **A**

27. $x=\int_0^2\left(t^3+\cos t\right)dt=\left.\dfrac{t^4}{4}+\sin t\right|_0^2\approx 4.9$ **E**

28. $f(x)=|x|=\begin{cases}x & x\ge 0\\ -x & x<0\end{cases}$ $f'(x)=\begin{cases}1 & x\ge 0\\ -1 & x<0\end{cases}$

$\lim\limits_{x\to 0^-}f'(x)=-1$ **A**

29. $x^2+x+6\big|_{x=2}=12$ $x-2\big|_{x=2}=0$ $\dfrac{12}{0}\to\infty$ (does not exist) **D**

30. $f(x)=2x^2-5x-3$ $f'(x)=4x-5$ $f'(2)=3$ **A**

1. $f'(x)=3x^2+5$ $f''(x)=6x$ $g'(x)=16x^3+2x$ $g''(x)=48x^2+2$

   $A=f'(1)=8$ $B=f''(2)=12$ $C=g'(0)=0$ $D=g''(-1)=50$

   $$\int_0^{50} x\cos(8x^2+12)dx=\left[\frac{1}{16}\sin(8x^2+12)\right]_0^{50}\approx .037$$

2. $$A=\frac{1}{2}\int_0^{\pi}(1-\cos 2x)dx=\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_0^{\pi}=\frac{\pi}{2}$$

   $$B=\int_1^2(u+1)\sqrt{u}du=\int_1^2\left(u\sqrt{u}+\sqrt{u}\right)du=\frac{2u^{5/2}}{5}+\frac{2u^{3/2}}{3}\Big|_1^2=\frac{44\sqrt{2}-16}{15}$$

   $$C=\frac{3^x}{\ln 3}\Big|_{-1}^{1}=\frac{8}{3\ln 3}\qquad D=\left[x\ln x-x\right]_1^e=1\qquad A\cdot B\cdot C\cdot D\approx 11.7$$

3. $9-x^2=x^2-9\Rightarrow x=\pm 3$

   $$V=\int_{-3}^{3}\frac{(2(9-x^2))^2\sqrt{3}}{4}dx=2\cdot\frac{4\sqrt{3}}{4}\int_0^3(x^4-18x^2+81)dx=2\sqrt{3}\left[\frac{x^5}{5}-6x^3+81x\right]_0^3=\frac{1296\sqrt{3}}{5}$$

4. $$A=\int_0^4 1.3^x dx=\frac{1.3^x}{\ln 3}\Big|_0^4\approx 7.0745$$

   $$B=\frac{4}{8}\left[f(0)+f(.5)+f(1)+f(1.5)+\ldots+f(3.5)\right]\approx 6.6206$$

   $$C=\frac{4}{8}\left[f(.5)+f(1)+f(1.5)+\ldots+f(3.5)+f(4)\right]\approx 7.5487$$

   $$D=\frac{4}{8}\left[f(.25)+f(.75)+f(1.25)+f(1.75)+\ldots+f(3.75)\right]\approx 7.0694$$

   $$A+B+C+D\approx 28.31$$

5. $$A=\frac{1}{-4x^4}\Big|_1^{\infty}=0+\frac{1}{4}=\frac{1}{4}\qquad B=\frac{1}{3}\left[\frac{\sin 2x}{2}+\tan x\right]_0^{\pi/4}=\frac{1}{2}$$

   $$C=-e^{-x}\Big|_{-\ln 2}^{0}=-(1-2)=1\qquad D=\frac{(\ln x)^2}{2}\Big|_1^{e^2}=\frac{4-0}{2}=2$$

6. $2(3w+w\cdot l+3l)=32\Rightarrow l=\dfrac{16-3w}{w+3}$

   $$V=3wl=\frac{3w(16-3w)}{w+3}=\frac{3(16w-3w^2)}{w+3}$$

   $$\frac{dV}{dw}=3\frac{(w+3)(16-6w)-(16w-3w^2)(1)}{(w+3)^2}=3\frac{-3w^2-18w+48}{(w+3)^2}=-9\frac{(w^2+6w-16)}{(w+3)^2}$$

   Extrema: $\dfrac{dV}{dw}=0\Rightarrow w^2+6w-16=(w+8)(w-2)=0\Rightarrow w=2$

$$V_{\max} = \left.\frac{3w(16-3w)}{w+3}\right|_{w=2} = 12$$

7. $$\int_{-\pi}^{8} f(x)dx = \int_{-\pi}^{0} \sin\frac{x}{3}dx + \int_{0}^{2}\left(x^2+5x\right)dx + \int_{2}^{4} 2^{-x}(32)\ln 2\,dx + \int_{4}^{8}\frac{dx}{x\ln 4}$$
$$= -3\left[\cos\frac{x}{3}\right]_{-\pi}^{0} + \left[\frac{x^3}{3}+\frac{5x^2}{2}\right]_{0}^{2} + 32\left[-2^{-x}\right]_{2}^{4} + \frac{1}{\ln 4}\left[\ln x\right]_{4}^{8}$$
$$= -\frac{3}{2}+\frac{38}{3}+6+\frac{1}{2} = \frac{53}{3}$$

8. m = distance from base of pole to man s = length of shadow Given: $\frac{dm}{dt} = 7$

Geometry: $\frac{s}{6} = \frac{s+m}{20} \Rightarrow s = \frac{3m}{7} \Rightarrow \frac{ds}{dt} = \frac{3}{7}\frac{dm}{dt} = 3$

9. $f'(x) = \ln x + 1$ $f''(x) = \frac{1}{x}$ $g'(x) = \frac{3^x(\cos x\ln 3 + \sin x)}{\cos^2 x}$

$A = f'(g(1)) = f'(3/\cos 1) \approx 2.714$ $B = f(g'(2)) = f\left(\frac{9(\ln 3\cos 2 + \sin 2)}{\cos^2 2}\right) \approx 74.17$

$C = f''(g(2))f''(9/\cos 2) = \frac{\cos 2}{9} \approx -.04624$ $D = g'(f'(3)) = g'(\ln 3 + 1) = 12.28$

$A\cdot B\cdot C\cdot D \approx -114$

10. $$W = \int_{0}^{2} v(t)dt = e^t\left(t^2-2t+2\right)\Big|_{0}^{2} = 2e^2 - 2$$

$X = v(2) = 4e^2$

$Y = v'(2) = e^t\left(t^2+2t\right)\Big|_{t=2} = 8e^2$

$Z: t^2+2t<0 \Rightarrow t(t+2)<0 \Rightarrow 0<t<2$ $Z = 2$

$W - X - Y + Z = -10e^2$

11. Critical points of f: $f'(x) = 3x^2 - 6 = 0 \Rightarrow x = \pm\sqrt{2}$

Critical points of g: $g'(x) = 12x^3 + 12x^2 - 24x = 0 \Rightarrow x = -2, 0, 1$

$f(-2) = 6$ $f(-\sqrt{2}) = 4\sqrt{2}+2$ $f(\sqrt{2}) = -4\sqrt{2}+2$ $f(2) = -2$

Max: $A = 4\sqrt{2}+2$ Min: $B = -4\sqrt{2}+2$

$g(-3) = 19$ $g(-2) = -40$ $g(0) = -8$ $g(1) = -13$ $g(2) = 24$

Max: $C = 24$ Min: $D = -40$

$$\frac{C+D}{A+B} = -4$$

12. $f'(x) = \sec x \tan x + 1$ $\quad g'(x) = \dfrac{\tan 2x}{2\sqrt{x}} + 2\sqrt{x}\sec^2 2x$

$f'(\pi/6) = \dfrac{5}{3}$ $\quad \dfrac{-1}{f'(\pi)} = -1$ $\quad g'(4\pi) = 4\sqrt{\pi}$ $\quad g\dfrac{-1}{f'(\pi/2)} = -\dfrac{1}{\sqrt{2\pi}}$

$P \cdot Q \cdot R \cdot S = \dfrac{10\sqrt{2}}{3}$

13. $\int f(x)dx = \dfrac{\sin \pi x}{\pi} + 2x^4 + 2^x + C$

$A = \int_{-1}^{0} f(x)dx = -\dfrac{3}{2}$ $\quad B = \int_{0}^{1} f(x)dx = 3$

$C = \dfrac{1}{2}\int_{-2}^{0} f(x)dx = -\dfrac{125}{8}$ $\quad D = \dfrac{1}{2}\int_{0}^{2} f(x)dx = \dfrac{35}{2}$ $\quad \dfrac{ABC}{D} = 4.017 \approx 4$

14. $L = \lim\limits_{x\to\infty} \dfrac{3x^2}{x^2} = 3$ $\quad M = f(0) = 1$ $\quad N = \lim\limits_{x\to -5} \dfrac{(3x-2)(x+5)}{(x+5)(x-2)} = \dfrac{17}{7}$

$O = \lim\limits_{x\to -\infty} \dfrac{3x^2}{x^2} = 3$ $\quad \dfrac{3(1)}{(17/7)(3)} = \dfrac{7}{17}$

15. $f'(x) = \dfrac{x(3^x \pi \sec^2 \pi x + 3^x \ln(3)\tan \pi x) - 3^x \tan \pi x}{x^2}$ $\quad f'(3) = \dfrac{3(27\pi + 0) - 0}{9} = 9\pi$

$g'(x) = \dfrac{(\sec \pi x - \pi x \sec \pi x \tan \pi x)\ln 2}{\sec^2 \pi x}$ $\quad g'(3) = \dfrac{(-1-0)\ln 2}{1} = -\ln 2$

$f'(3) - g'(3) = 9\pi + \ln 2$