

Question #1
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

1. Find the sum of all real solutions of the given equation : $5e^x + \frac{7}{e^x} = 8 + 4e^x$

Question #1
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

1. Evaluate : $\cos [\text{Arctan} (\frac{5}{12}) - 2 \text{Arcsin} (\frac{4}{5})]$

Question #1
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

1. Find the length of the arc from $t = 1$ to $t = 3$ for the graph parametrized by the following equations:

$$x = \frac{3}{2}t^2$$

$$y = \frac{1}{9}(6t + 1)^{\frac{3}{2}}$$

Question #2
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

2. Given : $z_1 = 6 + 8i$ and $z_2 = 2 + i$ ($i = \sqrt{-1}$)

Find the quotient $\frac{z_1}{z_2}$ expressed in the form $a + bi$.

Question #2
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

2. Find the shortest distance between the focus of the parabola $4y = x^2 - 12x + 16$ and the line with equation $3x + 4y = 7$.

Question #2
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

2. Water is being poured into an inverted conical cylinder (vertex is at the bottom) with a height of 6 feet and a radius at the surface of 18 feet. Water is being poured in at a rate of $4 \text{ ft}^3/\text{min}$. In t min , how fast is the water level rising when the water is 2 feet from the bottom of the container?

Question #3
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

3. Find the area of the region formed by the intersection of the graphs of the following inequalities:

$$x^2 + y^2 \leq 36$$

$$y \geq |x|$$

Question #3
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

3. Given: $\vec{A} = \langle -2, 1, 2 \rangle$ and $\vec{B} = \langle -5, 5, 7 \rangle$

Find the value of $\|\vec{C}\|$, where $\vec{C} = \vec{A} \times \vec{B}$.

Question #3
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

3. Given : $6x = y^2 - 2y + 1$

Find the area of the triangular region enclosed by the latus rectum of the curve and the two tangents to the curve at the endpoints of the latus rectum.

Question #4
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

4. Given : $f(x) = \frac{6x^2 - 24}{2x^2 + 4x - 16}$

There is a vertical asymptote at $x = A$.
There is a horizontal asymptote at $y = B$.
There is a hole in the graph at (C,D) .

Find the product ABCD.

Question #4
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

4. Given : (a,b) is the point of inflection of $f(x) = 3x^3 - 9x^2 + 15$

Find the value of $a + b$.

Question #4
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

4. Find the sum of all integral values of x for which the following series converges:

$$(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots$$

Question #5
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

5. Find the value of the constant term in the expansion of $(2x^2 + \frac{1}{x})^6$.

Question #5
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

5. Given : $f(x) = 4 \sin (2x - \frac{3\pi}{4}) + 3 \cos (2x - \frac{3\pi}{4})$

Let A equal the period of the function.

Let B equal the amplitude of the function.

Find the product AB.

Question #5
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

5. Evaluate :

$$\lim_{a \rightarrow \infty} \sum_{b=1}^a \frac{a^2 + b^2 + 2ab}{a^3}$$

Question #6
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

6. Eric wants to solve a linear system in x , y , and z using Cramer's rule, but loses the original problem before he copies down the denominator determinant. Justin tells Eric he can still solve the problem based on the numerator determinants D_x and D_z , which he has already written down. Given the information below, find the value of the denominator determinant.

$$D_x = \begin{vmatrix} 7 & 1 & 1 \\ 13 & -1 & 3 \\ -4 & 4 & 6 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 1 & 1 & 7 \\ 2 & -1 & 13 \\ -5 & 4 & -4 \end{vmatrix}$$

Question #6
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

6. Given : $\sum_{n=1}^{\infty} (2^{(-n+3)} \text{cis}(\frac{n\pi}{2}))$

If the value of this summation is expressed in the form $a + bi$ ($a, b \in \mathfrak{R}$; $i = \sqrt{-1}$), find the sum $a + b$.

Question #6
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

6. Evaluate the following integral:

$$\int_{-1}^1 \left[\left(e^{x^2} \sin\left(\frac{\pi x}{4}\right) + \frac{1}{1+x^2} + \frac{5x^4+5}{x^5+5x} \right) dx \right]$$

Question #7
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

7. Given the following sequence : 12 , a , b , 32

If the first 3 terms are in arithmetic progression and the last 3 terms are in geometric progression, find the product of all possible values of b.

Question #7
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

7. Let A equal the units digit of 3^{442} .
Let B equal the value of the digit X so that 31,742,63X,589 is divisible by 9.
Find the product AB.

Question #7
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

7. Given: $\frac{dy}{dx} = e^{3y}\cos^2x$ and $y(0) = -\frac{1}{3}$
Find the value of $y(\frac{\pi}{2})$.

Question #8
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

8. Find the distance between the center of the graph with equation $x^2 - 10x + 4y^2 + 72y + 345 = 0$ and the focus of the graph with equation $4y = x^2 - 4x - 20$.

Question #8
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

8. Given : $(x+3)(x+4)(x-2)(x-1) = 24$
Find the sum of all non-negative roots of this equation.

Question #8
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

8. Find the area enclosed by the graph of $r = 4 + 4 \cos \theta$.

Question #9
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

9. A single card is drawn from a standard deck of cards at the same time two fair coins are tossed. Find the probability that the at least one coin is heads and that the card is either a king or a heart.

Question #9
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

9. The following statements have a numerical "value" in parentheses beside each statement. If the statement is true, its "value" is the number next to it. If the statement is false, its "value" is 0. Find the sum of the "values" of all of the true statements.

The graph of $r = 4 \cos 3\theta$ is a rose with 6 petals. (12)

The eccentricity of the graph of $r = \frac{4}{2 - 3 \cos \theta}$ is $\frac{3}{2}$. (-7)

$r^2 = 16 \sec 2\theta$ is the graph of a lemniscate. (-19)

The slope of a line with equation $\theta = \frac{\pi}{6}$ is $\frac{\sqrt{3}}{3}$. (42)

The area of the polar graph $r = 6 \cos \theta$ is 36π . (2)

Question #9
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

9. Evaluate : $\lim_{y \rightarrow 2} \left(\left(\frac{y}{2} \right)^{\tan\left(\frac{\pi y}{4}\right)} \right)$

Question #10
Ciphering Theta Level
2000 Mu Alpha Theta National Convention

10. Find the sum of all distinct real values of x that satisfy the following equation:

$$(x^2 - 9x + 19)(x^2 + 2x - 8) = 1$$

Question #10
Ciphering Alpha Level
2000 Mu Alpha Theta National Convention

10. There are 8 blue, 3 red, and 6 yellow marbles in a bag. If 3 marbles are selected at random, without replacement, and it is known that the first marble is not blue, find the probability that the third marble is yellow.

Question #10
Ciphering Mu Level
2000 Mu Alpha Theta National Convention

10. Find the volume of the solid whose base is the interior of the ellipse $9x^2 + 16y^2 = 144$ and whose cross sections perpendicular to the x -axis are equilateral triangles.