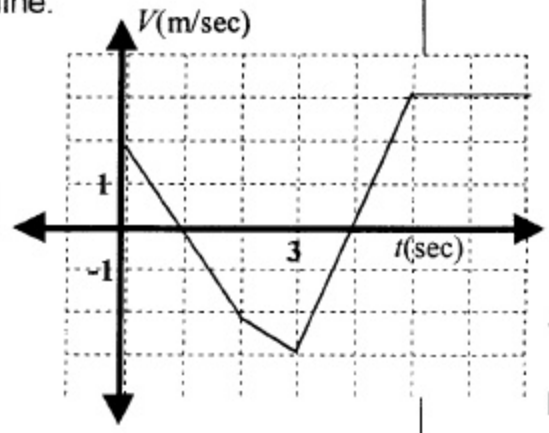
Integration – Mu Level
2000 Mu Alpha Theta National Convention

Choose NOTA if no other answer listed is correct.

1. Using the given diagram, estimate the total distance traveled by the particle. The graph shows the velocity of a particle moving along a horizontal line.



a. 3 meters
b. 7 meters
c. 13 meters
d. 21 meters
e. NOTA

2. Which integral has the same value as the given limit?

$$\lim_{n\to\infty} \frac{\sqrt{4}+\sqrt{8}+\ldots+\sqrt{4n}}{\sqrt{n^3}}$$

a. $\int_0^2 2\sqrt{x}\,dx$

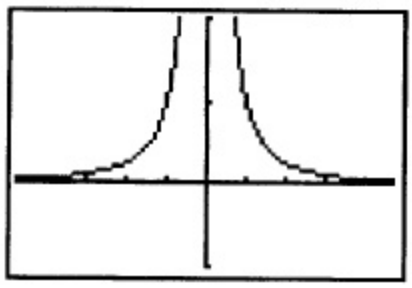
b. $\int_0^1 2\sqrt{x}\,dx$

c. $\int_0^1 2\frac{1}{\sqrt{x}}\,dx$

d. $\int_0^2 \frac{1}{\sqrt{x}}\,dx$

e. NOTA

3. The graph shown is the ***derivative*** of a function *f*. Which function could be the function whose derivative is shown in the window?



a. $f(x)=\frac{1}{x^2}$

b. $f(x)=\frac{1}{x}$

c. $f(x)=-\frac{1}{x}+2$

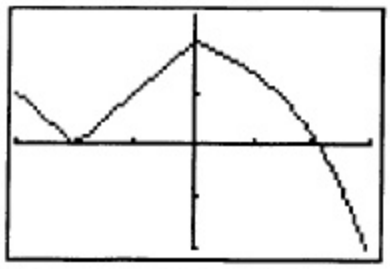
d. $f(x)=e^{|x+1|}$

e. NOTA

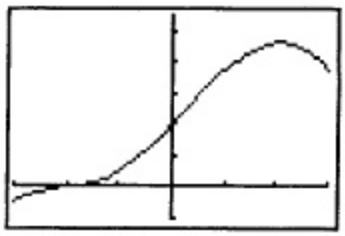
4. The price, P, in dollars per meter, for digging a mine is given by $P=80+0.03x^2$ where x is the distance in meters from the entrance to the mine. How much does it cost to dig 100 meters of this mine?

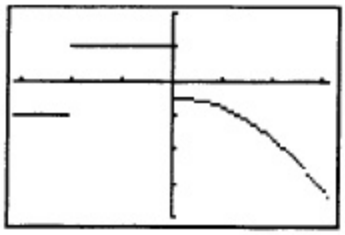
a. $9000
b. $15000
c. $18000
d. $20000
e. NOTA

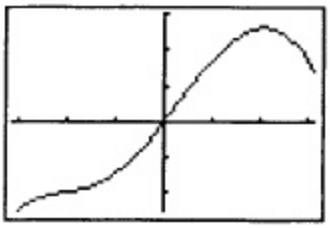
5. Let *f* be the function whose graph is shown below and let $g(x) = \int_{-2}^{x} f(x)dx$. Which of the following could be the graph of function *g*? (The scale is 1 unit between each mark on every graph that is shown.)

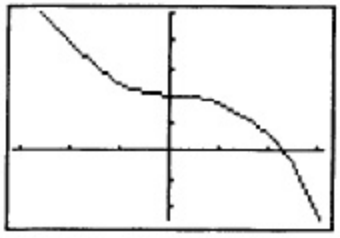


This is the graph of function *f*.

a. 

b. 

c. 

d. 

e. NOTA

6. Below is a Simpson's Rule approximation for a definite integral. Find a trapezoidal approximation (to the nearest thousandth) for the same integral using the same number of partitions.

$$\frac{1}{6}\left[0+4(2)\left(\frac{1}{2}\right)^2+2(2)(1)^2+4(2)\left(\frac{3}{2}\right)^2+2(2)^2\right]$$

a. 5.000
b. 5.250
c. 5.333
d. 5.500
e. NOTA

7. Let f be a function whose graph goes through the point (2,4) and whose derivative is given by $f'(x) = \sin(x^2)$. Use $\int_{2}^{2.3} f'(x)dx$ to approximate *f*(2.3).

a. 3.641
b. 3.658
c. 3.721
d. 4.024
e. NOTA

8. The rate at which water enters a pool, in gallons per hour, is given by a differentiable function W of time t. the table below shows the rate as meaaured every hour for an 8-hour period. Use Simpson's Rule with 8 subdivisions to approximate the total number of gallons that entered the pool during this time period.

t(hours)	*W(t)*
0	20
1	22.2
2	24
3	26.4
4	30
5	29.2
6	28.2
7	20
8	12

a. 192.833
b. 195.867
c. 200.067
d. 212
e. NOTA

9. Suppose f and g are continuous functions and that
$\int_3^4 f(x)dx = 9, \int_3^4 g(x)dx = 4,$ and $\int_0^4 g(x)dx = 6$
Which of the following must be true?

I. $\int_0^3 g(x)dx = 2$

II. $\int_3^4 [f(x)g(x)]dx = 36$

III. $\int_3^4 (f(x) - g(x))dx = 5$

a. I and II only
b. I and III only
c. I only
d. I, II, and III

10. Find the area of the region between the graph of
$y = 0.2x^2 - 0.8x - 1$ and the $x-$axis on $[-2,3]$.

a. 6
b. 14/3
c. 16/3
d. 32/3
e. NOTA

11. If integration by parts is used to evaluate $\int \sin^{-1}(2x)dx$, which of the following integrals could be obtained in the evaluation procedure?

a. $\int \frac{\sqrt{1-4x^2}}{2}dx$

b. $\int \frac{2x}{1+4x^2}dx$

c. $\int \frac{2}{\sqrt{1-4x^2}}dx$

d. $\int \frac{2x}{\sqrt{1-4x^2}}dx$

e. NOTA

12. If the half-life of a certain radioactive element is 0.5 years, how many years will it take for 98% of a sample to decay? (Round to the nearest thousandth.)

a. 2.822
b. 2.623
c. 2.214
d. 0.015
e. NOTA

13. Suppose Euler's method with increment Δx is used to approximate a solution to a differential equation $\frac{dy}{dx} = f(x,y)$ and (x_o, y_o) lies on the solution curve. Let (x_n, y_n) denote the points generated after n iterations of Euler's method. Let $y = y(x)$ denote the exact solution to the differential equation. Which of the following must be true?

I. $y_4 < y(x_4)$
II. $y_3 = y_2 + f(x_2, y_2) \cdot \Delta x$
III. $x_4 = x_o + 4\Delta x$

a. II only
b. I and II only
c. II and III only
d. I, II, and III
e. NOTA

14. A cylindrical hot tub is 10 feet in diameter and 4 feet deep. At $t = 0$ the drain is opened so that water will flow out. The rate at which it flows out is directly proportional to the square root of the volume of water in the tub at any time. If the tub was full of water initially, after how many minutes will it be empty if water flowed out initially at $10\pi \frac{\text{ft}^3}{\text{min}}$?

a. 10
b. 15
c. 18
d. 20
e. NOTA

15. Suppose a slope field is drawn for the differential equation $\frac{dy}{dx} = -\frac{0.4x}{y}$. The slope field would show which of the following?

a. a family of hyperbolas
b. a family of ellipses
c. a family of circles
d. a family of lines
e. NOTA

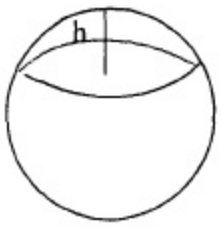
16. A spring obeys Hooke's Law and requires a force of 8N to stretch it 6 cm beyond its natural length. How much work is done in stretching it from its natural length to 12 cm beyond its natural length?

a. 16 $N \cdot cm$
b. 54 $N \cdot cm$
c. 64 $N \cdot cm$
d. 96 $N \cdot cm$
e. NOTA

17. Find the area of the larger region bounded by the graphs of $x = y^2 - 2y$ and $x = -y^3 + 2y + 4$.

a. 45/4
b. 27/4
c. 21/4
d. 15/4
e. NOTA

18. A sphere of radius *r* is sliced *h* units from its edge (*h* < *r*) to form a spherical cap. Find the volume of this cap in terms of *h* and *r*.



a. $h^2r - \frac{h^3}{3}$

b. $\frac{1}{6}\pi h(3r^2 + h^2)$

c. $\frac{\pi}{3}h^2(3r^2 - h)$

d. $\frac{\pi}{3}h^2(3r - h)$

e. NOTA

19. Find the volume of the solid generated when the region enclosed by $y = \cos x$ and the axes on $\left[0, \frac{\pi}{2}\right]$ is rotated about the y-axis.

a. $2\pi^2$

b. $2\pi^2 - 4\pi$

c. $2\pi - 4$

d. 2π

e. NOTA

20. Which of the following is the total arc length of the ellipse defined by $x = a\cos t$ and $y = b\sin t$ on $[0,2\pi]$ for $a > b > 0$?

a. $4a\int_0^{\frac{\pi}{2}} \sqrt{1 - k^2\cos^2 t}\,dt$ where $k^2 = \frac{a^2 - b^2}{a^2}$

b. $a\int_0^{2\pi} \sqrt{1 - k^2\sin^2 t}\,dt$ where $k = \frac{\sqrt{a^2 - b^2}}{a^2}$

c. $\int_0^{2\pi} \sqrt{a^2\cos^2 t + b^2\sin^2 t}\,dt$

d. $4\int_0^{\frac{\pi}{2}} \sqrt{a^2(a - \cos^2 t) - b^2\cos^2 t}\,dt$

e. NOTA

21. When evaluating $\int\sqrt{1 + e^x}\,dx$, the substitution $x = \ln(u^2 - 1), u > 1$, is made. Which of the following steps would be used in completing the evaluation of this integral?

a. $\sqrt{u^2} = u$

b. $\frac{2u^2}{u^2 - 1} = \frac{2}{u^2 - 1} + 2$

c. $u^2 - 1 = (u + 1)(u - 1)$

d. $\int\frac{u}{u^2 - 1}\,du = \frac{1}{2}\ln(u^2 - 1) + C$

22. The graph of $y = x^{\frac{1}{3}}$ on [0, 8] is rotated around the y-axis to form a solid. Which of the following integrals could be used to find its surface area?

a. $2\pi\int_0^8\left(x^{4/3}+\frac{1}{9}\right)^{1/2}\cdot x^{1/3}dx$

b. $2\pi\int_0^8\sqrt{1+\frac{1}{9}x^{-4/3}}dx$

c. $2\pi\int_0^8\sqrt{1+\frac{1}{9}x^{-1/3}}dx$

d. $\frac{3\pi}{2}\int_0^8 x\sqrt{x^{4/3}+\frac{1}{9}}dx$

e. NOTA

23. Express $\int_0^x\frac{1-\cos t}{t^2}dt$ as a power series. (NOTE: It is fine for this integral to be improper.)

a. $-\frac{1}{2}+\sum_{n=0}^{\infty}\frac{x^{2n}(-1)^n}{(2n+2)!}$

b. $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)(2n+2)!}$

c. $\sum_{n=0}^{\infty}\frac{x^{2n}(-1)^n}{(2n)!}$

d. $\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n-1}}{(2n-1)(2n)!}$

24. A leaky 10-pound bucket is lifted from the ground into the air by pulling in 20 feet of rope at a constant speed. The rope weighs 0.05 lb/ft. The bucket starts with 14 pounds of water and leaks at a constant rate. It finishes draining just as it reaches the top. How much work (in foot-pounds) was done lifting the water, the bucket, and the rope to this point 20 feet in the air?

a. 140
b. 340
c. 350
d. 375
e. NOTA

25. Find the volume of the solid whose base is the interior of the ellipse $x^2+4y^2=4$ and whose cross-sections perpendicular to the x-axis are semicircles.

a. $\frac{2\pi}{3}$ b. $\frac{4\pi}{3}$ c. $\frac{8\pi}{3}$
d. $\frac{16\pi}{3}$ e. NOTA

26. What is the average value of f on [0, k] if $f(x)=ax^2$?

a. $\frac{a}{3}k^3$

b. $\frac{a}{2}k^2$

c. ak^2

d. $3ak^2$

e. NOTA

27. The region under the graph of $y = \sin x$ on [0,1] is rotated about the x-axis. Which of the following represents the volume of this solid?

a. $\int_0^1 (\sin^2 x)dx$

b. $\pi\int_0^1 \sin x dx$

c. $2\pi\int_0^2 (x \sin x)dx$

d. $\pi\int_0^1 \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)dx$

e. NOTA

28. Suppose you are driving 60 ft/sec behind a car and decide to pass. You accelerate at $\frac{6}{\sqrt{t}}$ feet per second per second. If it took you 25 seconds to pass the car, how many feet did you travel in that time?

a. 1000
b. 1500
c. 2500
d. 3000
e. NOTA

29. A dam has a vertical cross-sectional shape of $y = x^2$ for $0 \le y \le 9$. The pressure at any point below the surface of water is directly proportional to its distance from the surface to the point. (The proportionality constant is 62.4 pounds per cubic foot.) Find the total force in pounds that is acting on the dam when the water is at the top of the dam.

a. 64.8
b. 129.6
c. 1036.8
d. 8087.04
e. NOTA

30. If trig substitution is used to evaluate $\int \frac{1}{\sqrt{x^2 - 9}}dx$, which integral below would be obtained?

a. $\int \frac{1}{3}(\cot\theta)d\theta$

b. $\int \frac{1}{3}(\tan\theta)d\theta$

c. $\int \sec\theta\tan\theta\, d\theta$

d. $\int (\sec\theta)d\theta$

e. NOTA