

January 1, 2010

Maximal High School Invitational

1. Find the absolute maximum value of f in the interval $[-3,3]$ if $f(x) = 2x^3 - 6x - 1$.

- A) -5 B) -1 C) 3 D) 37 E) NOTA

2. A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides at right angles to the sheet. How many inches should be turned up on each side to give the gutter its greatest capacity?

- A) 1 B) 1.5 C) 3 D) 3.5 E) NOTA

3. $f(x) = x^3 + cx^2 + dx + e$. For what values of c and d will f have critical values of $x = 4$ and $x = -2$?

- A) $c = -5, d = -8$ B) $c = -5, d = -12$ C) $c = -3, d = -24$
D) $c = \frac{-3}{2}, d = 0$ E) NOTA

4. In what interval is the value of f negative while the function is increasing if $f(x) = 2x \ln(x)$?

- A) $\left(0, \frac{1}{e}\right)$ B) $(0,1)$ C) $\left(\frac{1}{e}, e\right)$ D) $(1, e)$ E) NOTA

5. For what values of k will the graph of $y = x^3 + kx^2 + x + 2$ have two horizontal tangent lines?

- A) $k=3$ B) $-3 < k < 3$ C) $k < -3$ or $k > 3$
D) $|k| > \sqrt{3}$ E) NOTA

6. In the interval $[0, \pi]$ what is the x coordinate of the points of inflection for $y = \sin^2(x) - \cos^2(x)$?

- A) $0, \pi$ B) $\frac{\pi}{4}, \frac{3\pi}{4}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{3}, \frac{2\pi}{3}$ E) NOTA

7. A decreasing continuous curve has the property that the slope of the normal line at x is equal to the negative of the slope of the tangent line at $\frac{x}{3}$ for each x . What is the slope of the tangent line at $x = 0$?

- A) $-e$ B) 0 C) -1 D) $-\frac{1}{3}$ E) NOTA

8. If $s(t) = t^3 - 6t^2$ for $t \geq 0$ is the position on the x -axis of a particle at time t . Find the minimum velocity of the particle.

- A) 4 B) 0 C) -2 D) -12 E) NOTA

9. Find the dimensions of the rectangle of maximum area having two vertices on the x -axis and two vertices above the x -axis on the graph of $y = 4 - x^2$.

- A) $\frac{4}{3}x, \frac{20}{9}$ B) $\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$ C) $\frac{2\sqrt{3}}{3}x, \frac{8}{3}$
 D) $\frac{4\sqrt{3}}{3}, \frac{8}{3}$ E) NOTA

10. Find the x -coordinate of the absolute maximum of $y = x^2 - 6x + 8$ on $[0, 4]$.

- A) 0 B) 2 C) 3 D) 4 E) NOTA

11. What conditions on a , b , and c will make $f(x) = ax^3 + bx^2 + cx + d$ always increasing?

- A) $b^2 - 4ac < 0$ and $a > 0$ B) $b^2 < 3ac$ and $a > 0$ C) $b^2 < 3c$ and $a > 0$
 D) cannot be determined E) NOTA

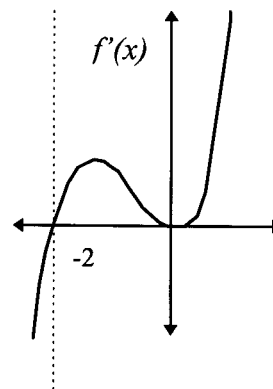
12. The acceleration of an object moving along a horizontal coordinate line is given by $a(t) = (2t + 3)^{-3}$. If the velocity at $t = 0$ is $\frac{35}{36}$ and the initial position is 1 on the coordinate line, which of the following best describes the location of the point when $t = 2$?

- A) at $\frac{3}{2}$ B) left of 1 C) left of 2 D) right of 2 E)

NOTA

13. The given figure shows the graph of f' , the derivative of f . Which of the following must be true?

- A) f' has an inflection at the origin
 B) f has a relative maximum at $x = 1$
 C) f has an inflection at $x = 0$
 D) f has a relative minimum at $(0, 0)$
 E) NOTA



14. What is the maximum area, in square yards, which can be inclosed by 12π yards of fencing?

- A) $9\pi^2$ B) 36π C) $36\pi^2$ D) 144π E) NOTA

15. A bomb is dropped from a plane which is 1600 feet above the ground. What is the vertical speed of the bomb (in feet per second) just before it hits the ground? [Neglect air resistance]

- A) -160 B) -320 C) -512 D) -1440 E) NOTA

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16. If $f(x)$ and $g(x)$ are both increasing functions, which of the following must be true?

- I. $f(x) + g(x)$ is an increasing function
- II. $(f(x))g(x)$ is an increasing function
- III. $f(g(x))$ is an increasing function

A) I only B) II only C) I & III only D) I, II & III E) NOTA

17. What positive number exceeds its cube by the greatest amount?

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{\sqrt{3}}{3}$ D) $\sqrt{3}$ E) NOTA

18. If $f''(x)$, the second derivative of $f(x)$, equals $(x+1)(x-1)^2(x-3)$, then which of the following is true?

- A) $f(x)$ has 2 inflections points and is concave up on the intervals $(-1,1)$ and $(3,+\infty)$
- A) $f(x)$ has 2 inflections points and is concave up on the intervals $(-\infty,-1)$ and $(3,+\infty)$
- A) $f(x)$ has 3 inflections points and is concave up on the intervals $(-1,1)$ and $(3,+\infty)$
- A) $f(x)$ has 3 inflections points and is concave up on the intervals $(-\infty,-1)$ and $(3,+\infty)$
- E) NOTA

19. Find the height, in inches, of the right circular cylinder of greatest lateral surface area that can be inscribed in a sphere of radius 12 inches.

A) $6\sqrt{2}$ B) $12\sqrt{2}$ C) $18\sqrt{2}$ D) $4\sqrt{3}$ E) NOTA

20. The derivative of $f(x) = \frac{x^4}{12} - \frac{x^5}{5}$ attains its maximum value at $x =$

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) 0 D) $-\frac{7}{180}$ E) NOTA

21. If $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$ then

- A) $f''(x) \geq 0$ for all x B) $f'(x) > 0$ for all $x > 0$
- C) $f(x)$ has three points of inflection D) $f'''(x)$ is constant
- E) NOTA

22. Given $P(p,4)$, $A(0,0)$, $B(8,3)$ and P is on the line $y = 4$. The value of p such that $PA+PB$ is a minimum is

A) 4 B) $6\frac{2}{5}$ C) $\frac{81}{16}$ D) $\frac{11}{2}$ E) NOTA

23. At 1 p.m. A is 34 miles due east of B and traveling at a constant speed of 10 mph due west. B is traveling due south at a constant speed of 6 mph. In how many hours is the distance between A and B minimum?

- A) $2\frac{1}{2}$ B) $\frac{170}{53}$ C) $3\frac{2}{5}$ D) $8\frac{1}{2}$ E) NOTA

24. Find the volume of the largest cone which can be inscribed in a sphere of radius 1.

- A) $\frac{32\pi}{81}$ B) $\frac{\sqrt{13}}{26}$ C) $\frac{\sqrt{13}}{13}$ D) $\frac{2\sqrt{13}}{13}$ E) NOTA

25. The abscissa of the first-quadrant point which is on the curve of $x^2 - y^2 = 1$ and closest to the point $(3,0)$ is

- A) 1 B) $\frac{3}{2}$ C) 2 D) 3 E) NOTA

26. If $f(x)$ is continuous and differentiable over $[-2,5]$ and $-4 \leq f'(x) \leq 3$ for all x in $(-2,5)$ then the greatest possible value of $f(5) - f(-2)$ is

- A) 7 B) 9 C) 15 D) 21 E) NOTA

27. What are the dimensions of a rectangular block of concrete, of maximum volume, that would fit under a parabolic tunnel that is 12 feet high, 20 feet long, and whose cross-section has the shape of the graph of $y = -x^2$. (All dimensions are stated in feet)

- A) $4 \times 8 \times 20$ B) $2 \times 4 \times 10$ C) $2 \times 4 \times 20$ D) $4 \times 8 \times 10$ E) NOTA

28. A company offers the following schedule of charges: \$30 per thousand for orders of 50,000 or less, with the charge per thousand decreased by 37.5 cents for each thousand above 50,000. Find the order which will make the company's receipts a maximum.

- A) 60,000 B) 62,500 C) 65,000 D) 66,250 E) NOTA

29. For the function $f(x) = -x^2 + 8x$, let A be the largest value of a and B be the smallest value of b for which $ax \leq f(x) \leq bx$ on the interval $0 \leq x \leq 4$. Find $4A + 3B$.

- A) 24 B) 32 C) 40 D) 48 E) NOTA

30. A rectangle is inscribed in a 5-12-13 right triangle such that the triangle's right angle is shared with the rectangle. What is the largest possible area of the rectangle?

- A) 15 B) 17 C) $21\frac{1}{8}$ D) $26\frac{2}{3}$ E) NOTA

Answer Key for MAXIMAL High School

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|------------------------------------|--|-------|
| 1. E answer is 35 | 11. B | 21. A |
| 2. C | 12. D | 22. A |
| 3. C | 13. C | 23. A |
| 4. E $\left(\frac{1}{e}, 1\right)$ | 14. B | 24. C |
| 5. D | 15. E velocity is -320
speed is 320 | 25. B |
| 6. B | 16. C | 26. D |
| 7. C | 17. C | 27. |
| 8. D | 18. D | 28. C |
| 9. D | 19. B | 29. C |
| 10. A | 20. B | 30. A |