

Algebra 1

Regional MARCH 2006

E) NOTA means None Of The Answers

1. Solve for x : $\frac{x+3a}{a} - \frac{x-2a}{3b} - 5 = 0$

- A) $2a$ B) $2a^2$ C) $a-8ab$ D) $\frac{4(6ab-2a^2)}{2b-a}$ E) NOTA

2. If $8^x = 128$ then $x =$

- A) 2 B) $\frac{7}{3}$ C) 3 D) $\frac{5}{3}$ E) NOTA

3. Factor : $b^3 - 3b^2 + 4b - 12$

- A) $(b^2 + 4)(b + 3)$ B) $(b - 3)^2(b^2 + 4)$ C) $(b^2 + 4)(b - 3)$ D) $(b + 2)(b - 2)(b - 3)$ E) NOTA

4. Simplify: $\frac{\sqrt{3}}{\sqrt{6} + \sqrt{3}}$

- A) $\frac{\sqrt{2} + 1}{3}$ B) $\sqrt{2} - 1$ C) $\sqrt{2} + 1$ D) $\frac{1}{\sqrt{2} + 1}$ E) NOTA

5. Given two numbers, if you subtract one-third the smaller number from each number, the result with the larger number is four times as great as the result with the smaller number. How many times as large of the smaller number is the large number?

- A) 3 B) $\frac{7}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) NOTA

6. Simplify: $\frac{[2(4^2 - 3^2)] + [6 + 4(1^2 - 6^0)]}{(2^3 \div 1^4) - [3^3 - 24]}$

- A) $\frac{24}{5}$ B) 4 C) $\frac{28}{33}$ D) -20 E) NOTA

7. The infinite repeating decimal .0525252... when expressed as a fraction is:

A) $\frac{13}{25}$

B) $\frac{52}{99}$

C) $\frac{26}{495}$

D) $\frac{11}{38}$

E) NOTA

8. If $a^3 - 3b = a^3 - 3b$ for all integers, then $3^3(2^3) =$

A) 4

B) 8

C) 12

D) 16

E) NOTA

9. The assignment was to reduce the given rational expressions, if possible. Steve tried them all but, which are right?

I. $\frac{x^2 - 4}{x + 2} = x - 2$

II. $\frac{2(x + y)}{2x + 2y} = 1$

III. $\frac{(x + 3) - x}{(x + 3)(x - 3)} = \frac{-x}{x - 3}$

A) I only B) II only C) II and III only D) I and II only E) NOTA

10. The slope of a line containing the points (7,12) AND (-4,8) is

A) $\frac{4}{11}$

B) $\frac{-4}{11}$

C) $\frac{11}{4}$

D) $\frac{-11}{4}$

E) NOTA

11. Which of the 4 numbers listed is the smallest number?

A) $\sqrt{5}$

B) $\frac{1}{5\sqrt{5}}$

C) $\frac{1}{\sqrt{5}}$

D) $\frac{\sqrt{5}}{5}$

E) NOTA

12. A stockbroker recommends that Mr. Holt invest in bonds and stocks at a ratio of 9:1, respectively. If Mr. Holt has \$37,000 to invest, how much should he invest in bonds? (Assume that Mr. Holt goes with the broker's recommendation.)

A) 3700

B) 37000

C) 25000

D) 33300

E) NOTA

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13. Megan has \$20 in dimes, quarters, and half-dollars. If she has 110 coins in all, and there are 2 fewer dimes than 6 times the half-dollars, how many quarters does she have?
- A) 28 B) 70 C) 12 D) 36 E) NOTA
14. Find the greatest common factor of 4320 and 1320
- A) 320 B) 120 C) 60 D) 10 E) NOTA
15. Write the equation of the line through the point (2,12) which is perpendicular to $2x + 8y = 5$.
- A) $4x - 3y = -28$ B) $4x - y = -4$ C) $3x - 4y = -42$ D) $x + y = 14$ E) NOTA
16. A survey of high school students' musical preference revealed the following data:
- | | | |
|---------------------------|---------------------------|--------------|
| 37 liked Ska | 62 liked Rap | 79 liked Pop |
| 12 liked only Rap and Pop | 11 liked only Ska and Pop | |
| 10 liked only Ska and Rap | 4 liked all three items | |
- How many students were surveyed? Assuming each liked at least one type music.
- A) 157 B) 135 C) 137 D) 142 E) NOTA
17. 16 is 0.5% of what number?
- A) 8 B) 320 C) 800 D) 3200 E) NOTA
18. Solve for x: $(4^2)(2^{x+2})(8^3) = 2^{15}$
- A) 2 B) 0 C) 1 D) -1 E) NOTA

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19. If $(x+y)^2 = 60$ and $(x-y)^2 = 40$; find the value of $x^2 + xy + y^2$
- A) 100 B) 55 C) 50 D) 20 E) NOTA
20. Rationalize: $\frac{2+\sqrt{2}}{2-\sqrt{2}}$
- A) -1 B) $2+3\sqrt{2}$ C) $3+2\sqrt{2}$ D) $2-\sqrt{2}$ E) NOTA
21. Simplify: $\frac{x^2+5x+6}{x^2+5x+4} \cdot \frac{x+1}{x-1} \div \frac{x^2+x-6}{x^2+3x-4}$
- A) 1 B) $\frac{x+2}{x-2}$ C) $\frac{x-1}{x+2}$ D) $\frac{x+2}{x-1}$ E) NOTA
22. Give the domain of the relation: $\{1,1\}, \{0,2\}, \{2,4\}$
- A) $\{0,1,2,4\}$ B) $\{0,1,2\}$ C) $\{1,2,4\}$ D) $\{2,2,6\}$ E) NOTA
23. Given that 4 trims are in 1 foot and 2.5 feet are in 1 lope. How many square trims are in 1 square lope?
- A) 10 B) 20 C) 25 D) 100 E) NOTA
24. $4x+(2y+3) = 4x+(3+2y)$ is an example of the _____ axiom
- A) Associative B) Commutative C) Transitive D) Reflexive E) NOTA
25. If a boat crew can row 15 miles upstream in 3.5 hours and 10 miles downstream in 75 minutes, how fast is the current?
- A) 1.5 mph B) $1\frac{6}{7}$ mph C) 2 mph D) 2.4 mph E) NOTA

26. Simplify : $\sqrt[3]{1^{15}2^{12}4^98^616^0}$
- A) 2^{10} B) 2^{16} C) 2^{18} D) 2^{21} E) NOTA
27. A tank can be filled by one pipe in 20 minutes and drained by another in 28 minutes. How long will it take to fill the tank if the two pipes are opened at the same time.
- A) $1\frac{1}{6}$ hrs B) $5\frac{1}{7}$ hrs C) 48 hrs D) 70 hrs E) NOTA
28. Find the sum of the roots of the equation: $2x^3 + 8x^2 - 7x^5 + 5x - 4$
- A) $\frac{2}{7}$ B) $\frac{-2}{7}$ C) -4 D) 0 E) NOTA
29. Phillip's age is $\frac{3}{4}$ of Pedro's. Seven years ago Pedro was 19 years younger than twice Phillip's age. Find their present ages.
- A) Phillip 39, Pedro 52 B) Phillip 56, Pedro 42
C) Phillip 32, Pedro 45 D) Phillip 84, Pedro 63
E) NOTA
30. If the instructions said to "evaluate" which of the following could be an answer?
- A) $y = 2x$ B) $x = 12$ C) $x < -5$ D) 12 E) NOTA

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Question 1 [A]

Common Denominator is $3ab$

$$\frac{3xb + 9ab}{3ab} - \frac{xa - 2a^2}{3ab} - \frac{15ab}{3ab} = 0$$

Group all terms with x

$$x(3b - a) + 9ab + 2a^2 - 15ab = 0$$

$$x(3b - a) + 2a^2 - 6ab = 0$$

$$x(3b - a) = 6ab - 2a^2 = 2a(3b - a)$$

$$x = 2a$$

Question 2 [B]

$$8 = 2^3 \text{ so } 8^x = (2^3)^x = 2^{3x}$$

$$128 = 2^7$$

$$2^{3x} = 2^7 \text{ so } 3x = 7$$

$$x = \frac{7}{3}$$

Question 3 [C]

$$b^3 - 3b^2 + 4b - 12$$

$$b^2(b - 3) + 4(b - 3)$$

$$(b^2 + 4)(b - 3)$$

Question 4 [B]

$$\frac{\sqrt{3}}{\sqrt{6} + \sqrt{3}} = \frac{\sqrt{3}}{\sqrt{6} + \sqrt{3}} \cdot \frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} - \sqrt{3}} = \frac{\sqrt{18} - 3}{6 - 3}$$

$$= \frac{3\sqrt{2} - 3}{3} = \sqrt{2} - 1$$

Question 5 [A]

Small number = S , Large number = L .

Subtract one third the small number from each number.

$$S - \frac{1}{3}S \text{ and } L - \frac{1}{3}S.$$

$$L - \frac{1}{3}S = 4(S - \frac{1}{3}S) \Rightarrow L - \frac{1}{3}S = 4S - \frac{4}{3}S$$

$$L = 4S - \frac{4}{3}S + \frac{1}{3}S = 4S - S = 3S \text{ or } L = 3S$$

Question 6 [B]

$$\frac{[2(16 - 9)] + [6 + 4(1 - 1)]}{(8 \div 1) - [27 - 24]} = \frac{[2(7)] + [6]}{8 - 3} = \frac{20}{5} = 4$$

Question 7 [C]

$$\frac{26}{495}$$

Question 8 [C]

$$2\gamma 1 = 2^3 - 3 \cdot 1 = 8 - 3 = 5$$

$$3\gamma 5 = 3^3 - 3 \cdot 5 = 27 - 15 = 12$$

Question 9 [D]

$$I. \frac{x^2 - 4}{x + 2} = \frac{(x - 2)(x + 2)}{(x + 2)} = x - 2$$

$$II. \frac{2(x + y)}{2x + 2y} = \frac{2(x + y)}{2(x + y)} = 1$$

$$III. \frac{(x + 3) - x}{(x + 3)(x - 3)} = \frac{3}{(x + 3)(x - 3)}$$

I and II.

Question 10 [A]

$$m = \frac{8 - 12}{-4 - 7} = \frac{-4}{-11} = \frac{4}{11}$$

Question 11 [B]

$$\frac{1}{5\sqrt{5}}$$

Question 12 [D]

Divide the \$37,000 into 10 parts. Each part is \$3,700. Invest 9 parts in bonds and 1 in stocks. So Mr. Holt must invest $9 \cdot 3700 = \$33,300$ in bonds.

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Question 13 [A]

$$d \cdot 0.10 + q \cdot 0.25 + h \cdot 0.50 = 20$$

$$d + q + h = 110$$

$$6h - 2 = d$$

$$6h - 2 + q + h = 110 \text{ or } 7h + q = 112 \text{ (Equation 1)}$$

$$0.6h - 0.2 + 0.25q + 0.5h = 20$$

$$1.1h + 0.25q = 20.2 \text{ (Equation 2)}$$

Solve system of Equations 1 and 2 and find

$$q = 28$$

Question 14 [B]

$$4320 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$$

$$1320 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 11$$

$$\text{Greatest Common Factor} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 120$$

Question 15 [B]

$$\text{Slope of } 2x + 8y = 5 \text{ is } -\frac{1}{4}$$

Slope of its perpendicular is 4

$$y - 12 = 4(x - 2) \Rightarrow y - 12 = 4x - 8$$

$$4x - y = -4$$

Question 16 [C]

Draw a Venn Diagram and solve to get 137

Question 17 [D]

$$16 = 0.005 \cdot x \Rightarrow x = 3200$$

Question 18 [B]

$$(2^4)(2^{x+2})(2^9) = 2^{15} \Rightarrow 2^{4+x+2+9} = 2^{15}$$

$$15 + x = 15 \Rightarrow x = 0$$

Question 19 [B]

$$(x + y)^2 - (x - y)^2 = 4xy = 60 - 40 = 20$$

$$\Rightarrow xy = 5$$

$$(x + y)^2 = x^2 + 2xy + y^2 = 60$$

$$x^2 + xy + y^2 = 60 - 5 = 55$$

Question 20 [C]

$$\frac{2 + \sqrt{2}}{2 - \sqrt{2}} \cdot \frac{2 + \sqrt{2}}{2 + \sqrt{2}} = \frac{4 + 4\sqrt{2} + 2}{4 - 2} = \frac{6 + 4\sqrt{2}}{2} = 3 + 2\sqrt{2}$$

Question 21 [B]

$$\frac{(x+3)(x+2)}{(x+4)(x+1)} \cdot \frac{(x+1)}{(x-1)} \cdot \frac{(x+4)(x-1)}{(x+3)(x-2)} = \frac{(x+2)}{(x-2)}$$

Question 22 [B]

Question 23 [D]

$$1 \text{ lope} = 2.5 \text{ feet} = 10 \text{ trims}$$

So 1 square lope has 100 square trims

Question 24 [B]

Commutative

Question 25 [B]

b = speed in still water

c = speed of current

$$3.5(b - c) = 15 \Rightarrow 3.5b - 3.5c = 15$$

$$1.25(b + c) = 10 \Rightarrow 3.5b + 3.5c = 28$$

$$7c = 13, c = \frac{13}{7} = 1\frac{6}{7}$$

Question 26 [B]

$$\sqrt[3]{1^{15} 2^{12} 4^9 8^6 16^0} = \sqrt[3]{2^{12} 4^9 8^6} = 2^4 4^3 8^2 = 2^4 2^6 2^6 = 2^{16}$$

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Question 27 [A]

$V = \text{volume}$

$$\text{Fillrate} = \frac{V}{20} \text{ per minute}$$

$$\text{Drainrate} = \frac{V}{28} \text{ per minute}$$

$$\text{Both open, net fill rate} = \frac{V}{20} - \frac{V}{28} = \frac{V}{70}$$

$$\text{Filltime} = \frac{V}{\frac{V}{70}} = 70 \text{ minutes} = 1\frac{1}{6} \text{ hours}$$

Question 28 [D]

Sum of roots = Coefficient of $x^4 = 0$

Question 29 [A]

Phillip = p

Pedro = q

$$p = \frac{3}{4}q \text{ and } 2(p - 7) - 19 = q - 7$$

$$2p - 14 - 19 = \frac{4}{3}p - 7 \Rightarrow \frac{2}{3}p = 26$$

$$\Rightarrow p = 39 \text{ and } q = 52$$

Question 30 [D]

QUESTION 1 Algebra 1 Regional March 2006

$x \oplus y = xy$ and $x \otimes y = x - y$ then find $[3 \oplus (12 \otimes 8)] \otimes [(2 \oplus 3) \otimes 5]$

QUESTION 2 Algebra 1 Regional March 2006

The product of 2 consecutive odd positive integers added to their sum is 287. What are the 2 integers?

QUESTION 3 Algebra 1 Regional March 2006

Solve for all real values of x:

$$x\sqrt{x^3} = \frac{x^x}{x}$$

QUESTION 4 Algebra 1 Regional March 2006

Evaluate $(1,000,000,000,001)^2 - (999,999,999,999)^2$

QUESTION 5 Algebra 1 Regional March 2006

Simplify :

$$\frac{2^{3x+4} - 2^{3x+3}}{2^{3x}}$$

QUESTION 6 Algebra 1 Regional March 2006

Simplify Round to the nearest ten thousandth: $5 + \frac{1}{5 + \frac{1}{5 + \frac{1}{5 + \frac{1}{5 + \dots}}}}$

QUESTION 7 Algebra 1 Regional March 2006

The denominator of a fraction exceeds the numerator by 8. If 2 is subtracted from the numerator, and the denominator is unchanged, the resulting fraction has a value of $\frac{7}{9}$. Find the original fraction.

QUESTION 8 Algebra 1 Regional March 2006

Given that $f(x) = x^2$ and $g(x) = x - 1$ find $f\left(g\left(f\left(g\left(\frac{1}{2}\right)\right)\right)\right)$

QUESTION 9 Algebra 1 Regional March 2006

A jogger left home and jogged 5 miles east and 12 miles south. The jogger rested and calculated his distance from home "as the crow flies". What was this distance?

QUESTION 10 Algebra 1 Regional March 2006

Find the sum of the slopes of the lines determined by each of the following:

A) $(4, -2), (-7, 9)$

B) $4x - 3y = 9$

C) $\frac{x}{4} + \frac{y}{8} = 5$

D) $3y = x + 1$

QUESTION 11 Algebra 1 Regional March 2006

How many successive 10% discounts will be necessary to lower the price of an item to below 50% of the original price?

QUESTION 12 Algebra 1 Regional March 2006

A boy at a bus stop learned that the bus would leave in 38 minutes. The boy ran at an average speed of 12 mph and reached home at the same time as the bus. If the bus traveled at an average speed of 50 mph, how far was the bus stop from the boy's home?

QUESTION 13 Algebra 1 Regional March 2006

If y varies inversely as the square of x and y is 8 when $x = \frac{1}{2}$, find $|x|$ if

$$y = \frac{1}{8}$$

QUESTION 14 Algebra 1 Regional March 2006

Factor completely :

$$2ax^2 + 2bx^2 + 2a^2x + 3abx + b^2x + a^2b + ab^2$$

QUESTION 15 Algebra 1 Regional March 2006

The coordinates of the points A, B, C, and D are :

A(4, -6) B(0, -3) C(-2, -5) D(0, 4)

x = The slope of line AD

y = The slope of line AB

z = The slope of line BC

v = The slope of line CD

Find $8(x + y - z + v)$

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Question 1

$$\begin{aligned} & [3 \oplus (12 \otimes 8)] \otimes [(2 \oplus 3) \otimes 5] \\ &= [3 \oplus 4] \otimes [6 \otimes 5] \\ &= [12] \otimes [1] \\ &= 11 \end{aligned}$$

Question 2

let the two integers be n and $(n+2)$.

$$n(n+2) + n + (n+2) = 287$$

$$n^2 + 2n + n + n + 2 = 287$$

$$n^2 + 4n + 2 = 287$$

$$n^2 + 4n - 285 = 0$$

$$n = -19 \text{ or } n = 15$$

Since we are told that the integers are positive, $n = 15$, $n + 2 = 17$

Question 3

$$x\sqrt{x^3} = \frac{x^4}{x}$$

Rewrite the equation:

$$x^{\frac{5}{2}} = x^{4-1}$$

$$\frac{5}{2} = x - 1$$

$$x = \frac{7}{2}$$

Question 4

$$a^2 - b^2 = (a+b)(a-b)$$

$$(1,000,000,000,001)^2 - (999,999,999,999)^2$$

$$= (2,000,000,000,000)(2)$$

$$= 4,000,000,000,000$$

Question 5

Rewrite problem:

$$\frac{2^{3i+4} - 2^{3i+3}}{2^{3i}} = \frac{2^{3i}(2^4 - 2^3)}{2^{3i}} = (2^4 - 2^3) = 16 - 8 = 8$$

Question 6

$$\text{Set } x = 5 + \frac{1}{5 + \frac{1}{5 + \frac{1}{5 + \dots}}}$$

$$x = 5 + \frac{1}{x}$$

$$x^2 = 5x + 1$$

$$x^2 - 5x - 1 = 0$$

$$x = 5.192582403567$$

$$x \approx 5.1926$$

Question 7

Let numerator be x

$$\frac{x}{x+8} \text{ is the fraction.}$$

$$\frac{x-2}{x+8} = \frac{7}{9}$$

$$9x - 18 = 7x + 56$$

$$2x = 74$$

$$x = 37$$

$$\text{Original fraction is } \frac{37}{45}$$

Question 8

$$f\left(g\left(f\left(g\left(\frac{1}{2}\right)\right)\right)\right) = f\left(g\left(f\left(-\frac{1}{2}\right)\right)\right)$$

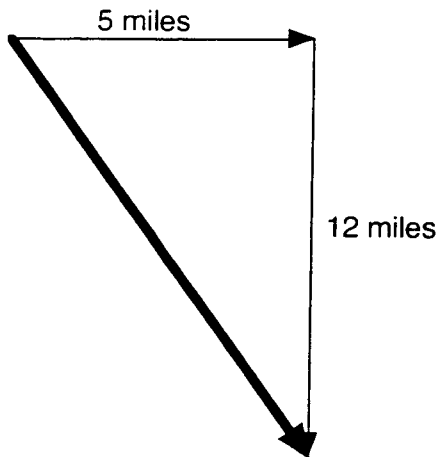
$$= f\left(g\left(\frac{1}{4}\right)\right) = f\left(-\frac{3}{4}\right) = \frac{9}{16}$$

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Question 9

Using the Pythagorean Theorem, the distance is 13 miles.



Question 10

$$A) \frac{9 - (-2)}{-7 - 4} = \frac{11}{-11} = -1$$

B) Rewrite in slope - intercept form to get slope = $\frac{4}{3}$

C) Rewrite in slope - intercept form to get slope = -2

D) Rewrite in slope - intercept form to get slope = $\frac{1}{3}$

$$\text{Sum of slopes} = -\frac{4}{3}$$

Question 11

Price after 10% discount is 0.9 times original price (P).

After x discounts, price is below 50% of original price (P).

$$(0.9)^x P < 0.5P$$

$$(0.9)^x < 0.5$$

$x = 6.57 \Rightarrow 7$ discounts are necessary

Question 12

Let distance be x miles.

Time taken by boy to run home = $\frac{x}{12}$ hours

Time taken by bus = $\frac{38}{60} + \frac{x}{50}$ hours

$$\frac{x}{12} = \frac{38}{60} + \frac{x}{50}$$

$$x = 10 \text{ miles}$$

Question 13

$$y = k \frac{1}{x^2}$$

$$8 = k \frac{1}{\left(\frac{1}{2}\right)^2}$$

$$k = 2$$

$$\frac{1}{8} = 2 \frac{1}{x^2}$$

$$|x| = 4$$

Question 14

$$\begin{aligned} &(a+b)(2x^2 + ab + 2ax + bx) \\ &= (a+b)[2x(a+x) + b(a+x)] \\ &= (a+b)(a+x)(2x+b) \end{aligned}$$

1

Question 15

$$x = \frac{4 - (-6)}{0 - 4} = -\frac{10}{4}$$

$$y = \frac{-3 - (-6)}{0 - 4} = -\frac{3}{4}$$

$$z = \frac{-5 - (-3)}{-2 - 0} = \frac{-2}{-2} = 1$$

$$V = \frac{4 - (-5)}{0 - (-2)} = \frac{9}{2}$$

$$8(x + y - z + V) = 2$$