

# Geometry



FELIX VARELA HIGH SCHOOL

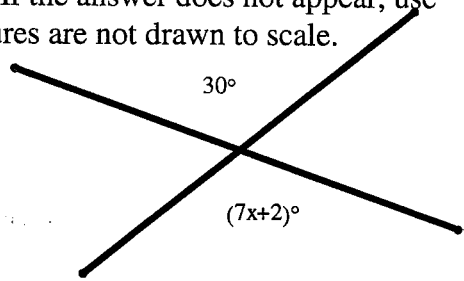
FAMAT REGIONAL COMPETITION

FEBRUARY 4, 2006

Directions: Select the best answer from the choices given. If the answer does not appear, use "E. NOTA", which stands for "None Of The Above". Figures are not drawn to scale.

1. Solve for x:

- A. -2      B. 1      C. 3  
 D. 4      E. NOTA

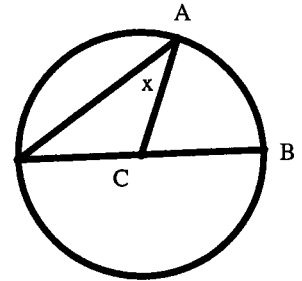


2. Which of the following pairs are NOT equivalent to each other.

- A.  $\overrightarrow{AB}, \overrightarrow{BA}$     B.  $\overline{AB}, \overline{BA}$     C.  $AB, BA$     D.  $\overline{AB}, \overrightarrow{BA}$     E. NOTA

3. On circle C, the measure of arc AB is  $30^\circ$ . Find the value of x:

- A.  $7.5^\circ$       B.  $15^\circ$       C.  $30^\circ$       D.  $45^\circ$       E. NOTA

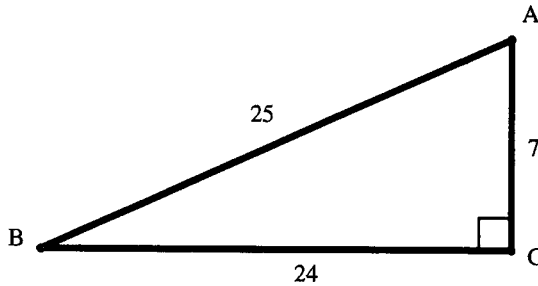


4. Two lines that never intersect are \_\_\_\_\_ defined as parallel lines. Fill in the blank.

- A. always    B. sometimes    C. never    D. historically    E. NOTA

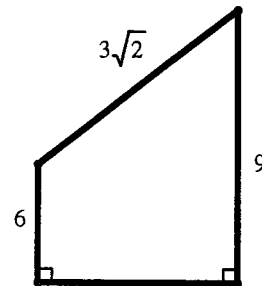
5. Find the value of  $\sin(A)$ :

- A.  $\frac{7}{24}$       B.  $\frac{7}{25}$       C.  $\frac{24}{25}$   
 D.  $\frac{25}{7}$       E. NOTA



6. Find the contrapositive of the statement. You win trophies if you practice hard.

- A. If you practice hard, then you win trophies.  
 B. If you don't win trophies, then you don't practice hard.  
 C. If you don't practice hard, then you don't win trophies.  
 D. If you win trophies, then you don't practice hard.  
 E. NOTA



7. Find the area of the figure at the right.

- A.  $\frac{45}{2}$       B. 27      C.  $\frac{45\sqrt{2}}{2}$       D.  $27\sqrt{2}$       E. NOTA

8. A cube has side length 5. Find the length of a diagonal of the cube.

- A. 25      B.  $2\sqrt{5}$       C.  $5\sqrt{2}$       D.  $5\sqrt{3}$       E. NOTA

9. Points A, B, and C are collinear.  $AB = 10$  and  $BC = 3$ . Which of the following is a possible value for AC?

A. 7      B. 9      C. 11      D. 15      E. NOTA

10. Five congruent spheres form a square pyramid. The four base spheres are tangent to two other base spheres and also tangent to the vertex sphere. The vertex sphere is tangent to all four base spheres. Find the height of the pyramid in terms of the radius,  $r$ .

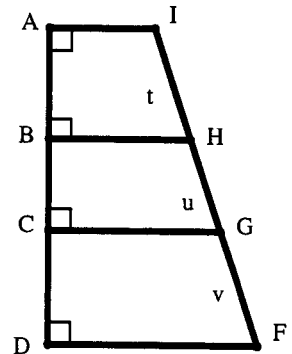
A.  $2r+r\sqrt{5}$     B.  $3r+r\sqrt{5}$     C.  $2r+r\sqrt{2}$     D.  $2r+r\sqrt{3}$     E. NOTA

11. A new system for the measurement of time divides one day into 20 decihours. Each decihour is made up of 100 deciminutes. Each deciminute is made up of 100 deciseconds. Twelve midnight is represented as 00:00:00 and twelve noon is represented as 10:00:00. Convert 07:72:94 from the new system to the traditional time. (Round to the nearest second.)

A. 8:22:34am      B. 9:16:31am      C. 9:27:53am  
 D. 12:52:12pm    E. NOTA

12. In the figure at the right,  $AB = 30$ ,  $BC = 35$ ,  $CD = 45$ , and  $IF = 88$ . Find  $t \times u \times v$ .

A. 88      B. 110      C. 24192  
 D. 47250    E. NOTA

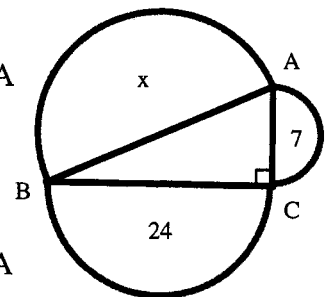


13. Which of the following could NOT be the sides of a triangle?

A. 5, 5, 8      B. 2, 7, 8      C. 5, 10, 9      D. 6, 10, 4      E. NOTA

14. In the figure at the right, 24 and 7 are areas of the semicircles. Find  $x$ , which is the area of the largest semicircle.

A. 25      B. 31      C. 37      D. 625      E. NOTA

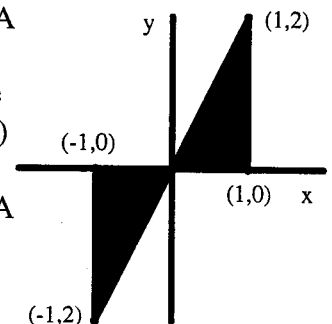


15. The diagonals of a cube divide the cube into how many congruent pyramids?

A. 2      B. 4      C. 6      D. 8      E. NOTA

16. The figure at the right is revolved around the y-axis. Find the surface area of the resulting 3D figure. (Round to the nearest whole number.)

A. 25      B. 32      C. 39      D. 46      E. NOTA



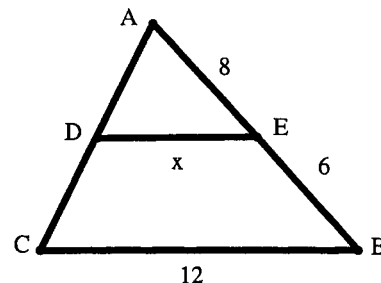
17. A rhombus is \_\_\_\_\_ a square. Fill in the blank.  
 A. always    B. sometimes    C. never    D. periodically    E. NOTA

18. In  $\triangle ABC$ ,  $m\angle A = x^\circ$ ,  $m\angle B = (x+1)^\circ$ ,  $m\angle C = 60^\circ$ . Which side is the longest?

- A.  $\angle B$     B.  $\overline{AC}$     C.  $\overline{BC}$     D.  $\overline{AB}$     E. NOTA

19. Given the figure at the right with  $\overline{DE} \parallel \overline{CB}$ , find x:

- A.  $\frac{36}{7}$     B.  $\frac{48}{7}$     C. 14    D. 16    E. NOTA

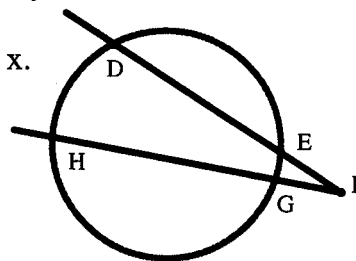


20. Points A, B, and C are distinct points located in the interior of a circle of radius 3. How many points are coplanar with A, B, and C AND equidistant from A and C AND 5 away from B?

- A. 1    B. 2    C. 4    D. infinitely many    E. NOTA

21. In the figure at the right,  $EF = 2$ ,  $DE = 6$ ,  $FG = 1$ , and  $GH = x$ . Find x:

- A. 11    B. 12    C. 15  
 D. 16    E. NOTA



22. Two similar pyramids have surface areas of 10 and 17. If the larger pyramid has a volume of 13, find the volume of the smaller pyramid.

- A.  $\frac{130\sqrt{170}}{289}$     B.  $\frac{13\sqrt{170}}{17}$     C.  $\frac{130}{17}$     D. Impossible    E. NOTA

23. A polygon with exactly one diagonal is a \_\_\_\_\_. Fill in the blank.

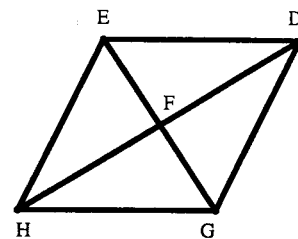
- A. figure    B. triangle    C. quadrilateral    D. square    E. NOTA

24. A cylinder of height 10 and base radius 2 contains water. The cylinder is placed on a flat horizontal surface so that it lies on a lateral edge (the circular bases are perpendicular to the flat horizontal surface). The water level has a height of 1. Find the volume of the water. (Round to the nearest tenth.)

- A. 24.0    B. 24.2    C. 24.4    D. 24.6    E. NOTA

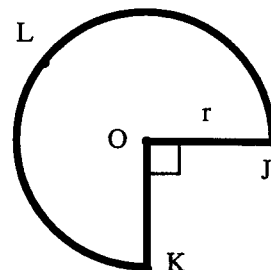
25. In parallelogram EDGH,  $HF = y^2 - 30$ ,  $FD = y$ ,  $EF = 3x$ , and  $FG = x + 40$ . Find  $x + y$ .

A. 15      B. 16      C. 25      D. 26      E. NOTA



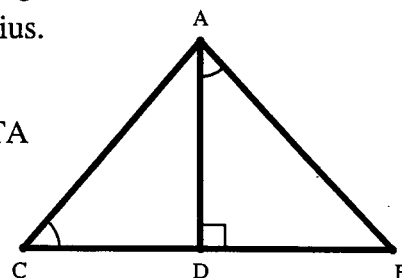
26. Consider figures in a plane: Given that points A and B are endpoints of a segment, find the locus of points that would form a right triangle with points A and B such that  $\overline{AB}$  is a leg.

A. a semicircle      B. a circle      C. a line  
D. two parallel lines      E. NOTA



27. The figure at the right is cut out along  $\overline{JK}$  and along radii  $\overline{OK}$  and  $\overline{OJ}$ . The cut out piece is then curved so that  $\overline{OK}$  and  $\overline{OJ}$  coincide, resulting in the lateral surface of a cone. Find the volume of the cone in terms of  $r$ , the radius.

A.  $\frac{4r^3\sqrt{\pi}}{9}$       B.  $\frac{4\pi^2r^3}{21}$       C.  $\frac{3\pi r^3\sqrt{7}}{64}$       D.  $\frac{15\pi r^3}{64}$       E. NOTA



28. Which of the following congruence methods would be used to show  $\triangle ABD \cong \triangle CAD$ ?

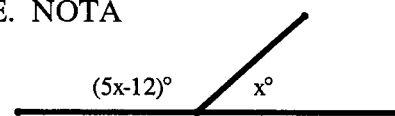
A. AAA      B. ASA      C. SAS      D. SSS      E. NOTA

29. A triangle has sides of length  $3t$ ,  $5t - 12$ , and  $t + 20$ . Find the sum of the values of  $t$  that would make the triangle equilateral.

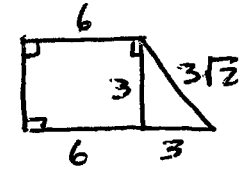
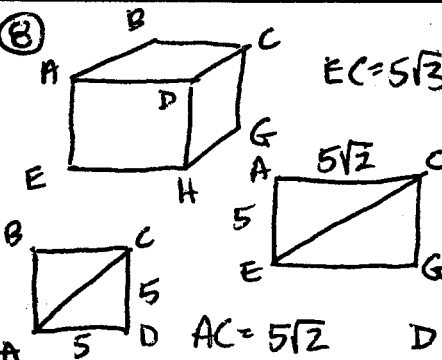
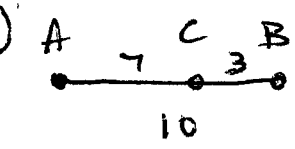
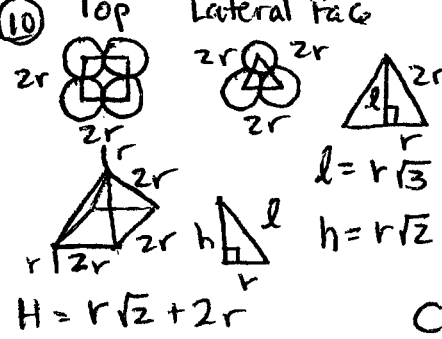
A. 6      B. 16      C. 18      D. 24      E. NOTA

30. Given the figure at the right, solve for  $x$ .

A. 2      B. 3      C. 30      D. 32      E. NOTA



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<p>① vertical <math>\angle</math>'s <math>\cong</math>  <math>30 = 7x + 2</math>  <math>4 = x</math></p> <p style="text-align: right;">D</p>	<p>⑥ if PH, then WT          Contrapositive          if <math>\neg</math>WT, then <math>\neg</math>PH</p> <p style="text-align: right;">B</p>	<p>⑪ <math>\frac{7:72:94}{10:00:00} = \frac{x}{12:00:00}</math>  <math>x = 9.27528</math>  <math>.27528 \times 60 = 16.5168</math>  <math>.968 \times 60 = 31.008</math>          9:16:31 am</p> <p style="text-align: right;">B</p>
<p>② <math>\overrightarrow{AB}</math> <math>\overrightarrow{A \rightarrow B}</math>  <math>\overrightarrow{BA}</math> <math>\overleftarrow{A \rightarrow B}</math></p> <p style="text-align: right;">D</p>	<p>⑦ <math>A = \frac{1}{2}(b_1 + b_2)h</math></p>  <p><math>A = \frac{1}{2}(15) \cdot 3</math></p> <p style="text-align: right;">A</p>	<p>⑫ AD is the shortest distance between lines <math>\overleftrightarrow{AE}</math> &amp; <math>\overleftrightarrow{DF}</math>          IF = 88 impossible</p> <p style="text-align: right;">E</p>
<p>③ <math>m\widehat{AB} = 30^\circ</math>  <math>\Rightarrow m\angle ACB = 30^\circ</math>          isosceles <math>\triangle ACD</math>  <math>\Rightarrow 2x = 30^\circ</math>  <math>\Rightarrow x = 15</math></p> <p style="text-align: right;">B</p>	<p>⑧</p>  <p><math>EC = 5\sqrt{3}</math>  <math>AC = 5\sqrt{2}</math></p> <p style="text-align: right;">D</p>	<p>⑬ <math>6 + 4 \neq 10</math></p> <p style="text-align: right;">D</p>
<p>④ skew lines</p> <p style="text-align: right;">B</p>	<p>⑨</p>  <p style="text-align: right;">A</p>	<p>⑭ <math>a^2 + b^2 = c^2</math>  <math>7 + 24 = 31</math></p> <p style="text-align: right;">B</p>
<p>⑤ <math>\sin(A) = \frac{24}{25}</math></p> <p style="text-align: right;">C</p>	<p>⑩ Top Lateral Face</p>  <p><math>H = r\sqrt{2} + 2r</math></p> <p style="text-align: right;">C</p>	<p>⑮ 6 congruent square pyramids, each w/ vertex = center of cube &amp; base = face of cube</p> <p style="text-align: right;">C</p>

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⑩  $SA = LA_{cyl} + 2LA_{cone}$   
 $LA_{cyl} = ph = 2\pi(1)(4)$   
 $LA_{cone} = \frac{1}{2}lp = \frac{1}{2}\sqrt{5}(2\pi)$   
 $SA = 8\pi + 2\sqrt{5}\pi$   
 $\approx 39.182$

⑪  $EF \cdot ED = FG \cdot FH$   
 $2 \cdot 8 = 1(1+x)$   
 $15 = x$

⑫

⑬ quadrilaterals  
 kites      trapezoids  
 rhombuses      parallelograms  
 rectangles  
 squares

⑭ Big Small  
 Length  $\lambda \sqrt{\frac{10}{17}}$   
 Area  $17 \times \frac{10}{17} = 10$   
 Volume  $13 \times \left(\sqrt{\frac{10}{17}}\right)^2 = \square$

⑮  $V = \frac{1}{3}Bh$    
 $P = \frac{3}{2}\pi r = 2\pi R$   
 $R = \frac{3}{4}$        $B = \pi\left(\frac{3}{4}r\right)^2$   
 $r^2 = h^2 + R^2$        $V = \frac{1}{3}\pi \frac{9}{4}r^2 \frac{r}{4}$   
 $h = \sqrt{r^2 - \left(\frac{3}{4}r\right)^2}$   
 $h = \sqrt{\frac{16}{16}r^2 - \frac{9}{16}r^2} = \frac{5}{4}\sqrt{7}$

⑯  $m\angle A = 59.5$   
 $m\angle B = 60.5$   
 $m\angle C = 60$   
 longest segment is  
 opposite  $\angle B$   
 $\overline{AC}$

⑰ # sides | # diagonals  

3	0
4	2
5	5

  
 Does not exist.

⑱   
 not necessarily  $\cong$

⑲  $\frac{x}{12} = \frac{8}{14}$   
 $x = \frac{8 \cdot 6 \cdot 2}{7 \cdot 2}$   
 $= \frac{48}{7}$

⑳  $Vol = A_{segment} \times 10$   
  
 $A = \frac{1}{2}(1)(2\sqrt{3})$   
 $A_{segment} = \frac{1}{3}(\pi(2^2) - \sqrt{3})$   
 $Vol \approx 24.567$

㉑ 

t	8	6	10
3t	24	18	30
5t-12	28	18	38
t+20	28	26	30

  
 not possible

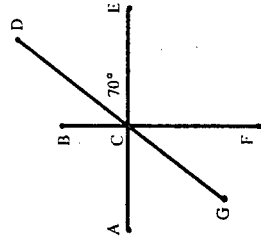
㉒

㉓  $HF = FD$        $EF = FG$   
 $y^2 - 30 = y$        $3x = x + 40$   
 $y = 6, -5$        $x = 20$   
 $x + y = 26$

㉔ Supplementary  
 $5x - 12 + x = 180$   
 $6x = 192$   
 $x = 32$

Question #1

How many of the following statements about the given figure can be assumed to be true?

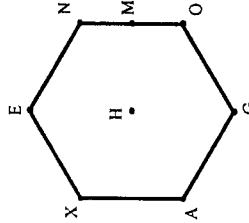


1.  $m\angle BCD = 20^\circ$
2. G, C, and D are collinear
3.  $m\angle ACG = 70^\circ$
4.  $m\angle FCD = m\angle BCG = 160^\circ$
5.  $\angle BCA$  and  $\angle FCA$  are supplementary
6.  $\overline{AE} \perp \overline{FB}$
7. A, B, and D are collinear
8.  $\overline{CG}$  bisects  $\angle ACF$
9.  $\angle BCA$  and  $\angle FCA$  are complementary
10.  $\angle GCA$  and  $\angle FCG$  are complementary
11.  $m\angle ACD = m\angle ECG = 110^\circ$
12.  $\angle BCA$  and  $\angle FCE$  are vertical angles

Question #2

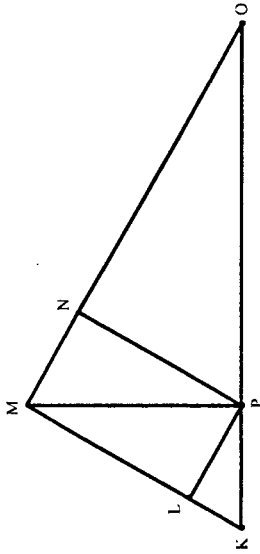
In the regular hexagon with center H, M is the midpoint of  $\overline{ON}$  and  $EN = 5$ .

Find  $\frac{EG + AE + XM + HM}{HA}$ , rounded to the nearest tenth.



Question #3

In  $\triangle KMO$ ,  $\overline{KM} \perp \overline{MO}$ ,  $\overline{IP} \parallel \overline{MO}$ ,  $\overline{KM} \parallel \overline{PN}$ ,  $\overline{PM} \perp \overline{KO}$ ,  $MP = 4$ , and  $KP = 3$ .



Find  $IM + NO + PO$ .

Question #4

Let the following list of numbers represent sets of lengths of segments.

1. 0.75, 1, 1.2
2.  $\frac{\pi}{2}, \frac{2}{\pi}, \pi$
3. 6, 6, 10
4. 8, 8, 16
5. 19, 23, 29
6.  $\sqrt{7}, \sqrt{11}, \sqrt{17}$
7. 3, 4, 5
8. 21, 220, 221

Let O = the number of obtuse triangles formed by the list of numbers.  
 Let A = the number of acute triangles formed by the list of numbers.  
 Let R = the number of right triangles formed by the list of number.

Find  $(O + A) \times R$ .

Question #5

Question #5

Let A meters = the perimeter of a square whose area is 100 square meters  
 Let B  $\pi$  square meters = the area of a circle whose circumference is  $8\pi$  meters  
 Let C square meters = the area of a rhombus with diagonals of length A meters and B meters  
 Let D square meters = the area of a trapezoid with height C meters and bases A meters and B meters.

Find  $A + B + C + D$ .



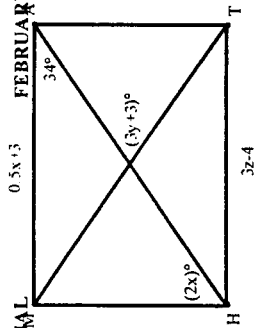
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Question #6

Given rectangle MATH, find  $x + y + z$ .



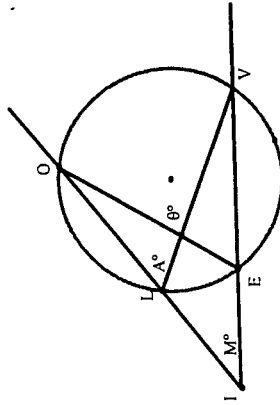
Question #7

Given 3 circles with radius, 5, find the sum of the areas of an inscribed equiangular triangle, an inscribed square, and an inscribed regular hexagon. (Round your answer to the nearest tenth.)

Question #8

If the figure,  $m\widehat{OY} = 100^\circ$  and  $m\widehat{LE} = 40^\circ$ .

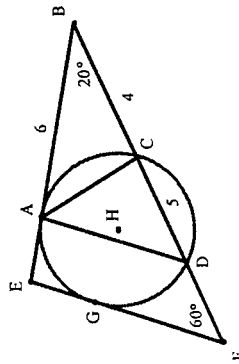
Find  $M + A + \theta$ .



Question #9

In the figure,  $\overline{EF} \parallel \overline{AD}$  and  $\odot H$  is tangent to  $\overline{EF}$  and  $\overline{BE}$  at points G and A, respectively.

Find  $m\angle DAC + m\angle ACB$ .



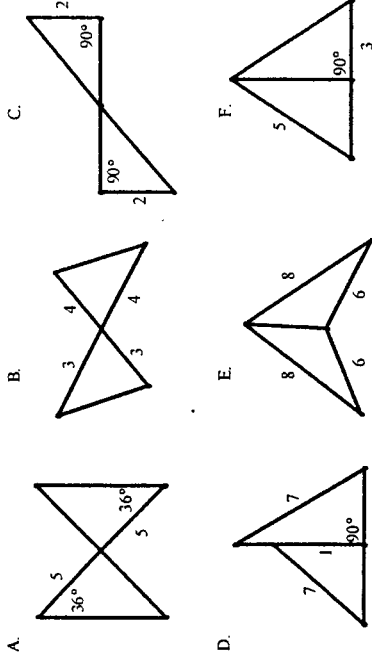
Question #10

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How many of the following figures represent pairs of triangles that must be congruent?



Question #11

Two congruent solid spheres of radius, 7, are melted down and recast into a cube and a square pyramid with base edge length, 7. Find sum of the length of a side of the cube and the height of the pyramid. (Round your answer to the nearest tenth.)

Question #12

Points O, A, B, and C lie on a number line with coordinates 0, 8, 12, and 26, respectively. Let P be any point not on the number line. PA is drawn and its midpoint labeled Q. Segment QB is drawn and its midpoint labeled R. Segment PC is drawn and its midpoint labeled S. What is the coordinate of the point where SR intersects the number line?

Question #13

Find the total number of faces on the 5 regular polyhedra (also known as Platonic solids).

Question #14

How many of the following are triangle congruence properties?

SSS, SAS, SSA, ASA, AAS, AAA

Question #15

How many of the following transformations always preserve distance?

Reflection, Rotation, Translation, Dilation, Glide Reflection

1. 4

1 – false; 2 – true; 3 – true; 4 – false; 5 – true; 6 – false; 7 – false; 8 – false; 9 – false; 10 – false; 11 – false; 12 – true

2. 6.4

HA = s so the sum divided by HA cancels out the common factor of s.

$$EG = 2; AE = \sqrt{3}; XM = \frac{\sqrt{13}}{2}; HM = \frac{\sqrt{3}}{2}.$$

3. 12.8

All triangles are similar to 3-4-5 right triangle. Set up extended proportions and solve.

$$x = 16/5; y = 64/15; z = 16/3$$

3		5	
4	x		y
5	4	3 + z	z

4. 8

Use triangle inequality postulate to make sure all sets form triangles. Check for right triangles. The rest are either acute or obtuse. Doesn't matter which since you are taking the sum.

2 – no triangle 4 – no triangle; 7 – right; 8 – right;

5. 9336

Square with area 100 has side 10 and perimeter 40. A = 40.

Circle with perimeter  $8\pi$  has radius 4 and area  $16\pi$ . B = 16.

Rhombus with diagonals 40 and 16 has area 320. C = 320.

Trapezoid with height 320 and bases 40 and 16 has area 8960.

So  $A + B + C + D = 40 + 16 + 320 + 8960 = 9336$ .

6.  $\frac{170}{3} = 56\frac{2}{3} = 56.\bar{6}$

$2x = 56, x = 28; 0.5*(28)+3 = 3z - 4, z = 7; 3y+3 = 68, y = 21\frac{2}{3}$

7. 147.4

Area triangle =  $\frac{1}{2} \cdot \frac{3}{2} \cdot 5 \cdot 5\sqrt{3}$  Area of square =  $(5\sqrt{2})^2$  Area of hexagon =  $6 \cdot \left(\frac{5^2\sqrt{3}}{4}\right)$

8. 150

$M = \frac{1}{2}(100-40); A = \frac{1}{2}(100); \theta = \frac{1}{2}(100+40)$

9. 140

by corr's angles  $m\angle ADC = 60^\circ$ ; by triangle sum  $m\angle E = 100^\circ$ ; by same side interior angles  $m\angle EAD = 80^\circ$ ; by  $\triangle BAC \sim \triangle BDA \sim \triangle BFE$   $m\angle BAC = 60^\circ$  and  $m\angle ACB = 100^\circ$ ;  $m\angle DAC = 40^\circ$

**10. 3**

A – ASA; B – no; C – AAS; D – no (1, 1, 7 does not form a triangle); E – SSS; F – no

**11. 99.2**

Volume sphere =  $\frac{4}{3}\pi(7)^3$ ; volume of cube =  $s^3$ ; volume of pyramid =  $\frac{1}{3}(7 \cdot 7)h$

$$s = 7\sqrt[3]{\frac{4\pi}{3}}, h = \frac{\frac{4}{3}\pi(7)^3}{\frac{1}{3}(7)^2}$$

**12. 3**

Since P can be any point not on the number line (x-axis), choose a point on the y-axis (0,20). Use the midpoint formula to find that Q is (4,10), R is (8,5), and S is (13,10). Use R and S to find the equation of the line and solve for  $y = 0$  to find the x-intercept (the coordinate that the ray intersects the original number line).

**13. 3**

reflection – no; rotation – yes; translation – yes; dilation – yes; glide reflection – no

**14. 4**

YES - SSS, SAS, ASA, AAS; NO - SSA, AAA

**15. 4**

Yes - Reflection, Rotation, Translation, Glide Reflection; No - Dilation

## Answers

## Individual

1. D
2. D
3. B
4. B
5. C
6. B
7. A
8. D
9. A
10. C
11. B
12. E
13. D
14. B
15. C
16. C
17. B
18. B
19. B
20. B
21. C
22. A
23. E
24. D
25. D
26. D
27. C
28. E
29. E
30. D

## Team

1. 4
2. 6.4
3. 12.8
4. 8
5. 9336
6.  $\frac{170}{3} = 56\frac{2}{3} = 56.\overline{6}$
7. 147.4
8. 150
9. 140
10. 3
11. 99.2
12. 3
13. 3
14. 4
15. 4