January Regional Answers

Individual

1 C
2 D
3 B
4 D
5 A
6 C
7 A
8 D
9 B
10 C
11 B
12 A
13 A
14 D
15 A
16 C
17 B
18 B
19 A
20 D
21 C
22 E
23 A
24 A
25 A
26 D
27 C
28 B
29 B
30 C

Team

1) 85
2) 10
3) 15
4) 9
5) 3
6) -1
7) 107
8) $\frac{10\sqrt{6}}{3}$
9) .17
10) 196
11) 13
12) 512
13) 1250
14) $28\sqrt{3}$
15) 8

NOTA means "None of the Above."

1. A triangle ABC has side lengths AB = 3, BC = 4, and AC = 5. Give the sum of the measures of the angles of this triangle.

   A) 90° B) 100° C) 180° D) 200° E) NOTA

2. Which of the following sets of numbers could not be the sides of a right triangle?

   A) $1, \sqrt{2}, \sqrt{3}$ B) 3, 4, 5 C) $1, 1, \sqrt{2}$ D) 1, 2, 3 E) NOTA

3. What is the area of an isosceles right triangle with a leg of length $\frac{1}{2}$ unit?

   A) $\frac{1}{4}\ un^2$ B) $\frac{1}{8}\ un^2$ C) $\frac{1}{16}\ un^2$ D) $2\ un^2$ E) NOTA

4. What is the circumference of a circle with an area of $4\pi\ m^2$?

   A) $2\pi\ m$ B) $4\ m$ C) $\pi\ m$ D) $4\pi\ m$ E) NOTA

5. For a positive integer x, a 3-sided figure has side lengths of $2x-7$, $4x+2$, and $5x+9$. For what values of x is the figure NOT a triangle?

   A) $x<14$ B) $x>14$ C) $x<13$ D) $x>13$ E) NOTA

6. Given ΔXYZ in the Cartesian Plane with X(2, 5), Y(-2, 3), Z(-2, -5), where does the median from the vertex at X intersect side YZ?

   A) (0, 0) B) (0, 4) C) (-2, -1) D) (-4, -2) E) NOTA

7. The point (2, -5) is in what quadrant?

   A) IV B) III C) II D) I E) NOTA

8. A circle, a square, and an equilateral triangle are all drawn with a pen. Which of the three figures takes the most amount of ink to draw, given the fact that they all have the same area?

   A) They all take the same amount.
   B) The circle.
   C) The square.
   D) The triangle.
   E) NOTA

9. Which of the following lines is the perpendicular bisector of the line segment with endpoints (9, -1) and (-3, -1)?

A) y = 3 B) x = 3 C) y = 6 D) x = 6 E) NOTA

10. From the following two statements, pick a valid conclusion: p, $p \rightarrow q$.

A) $q \rightarrow p$ B) $p \vee q$ C) q D) $\sim p$ E) NOTA

11. A 10 meter chord in circle O is 2 meters away from the diameter of the circle. What is the area of the circle, to the nearest square meter?

A) 66 B) 91 C) 236 D) 327 E) NOTA

12. A square has twice the area of another square. Find the ratio of the length of the side of the larger square to the length of the smaller square.

A) $\sqrt{2}:1$ B) 2:1 C) 1:2 D) 4:1 E) NOTA

13. If two parallel lines are cut by a transversal, then which of the following pairs of angles are <u>always</u> equal?
I. alternate interior II. alternate exterior III. same side interior

A) I, II B) I only C) II, III D) I, II, III E) NOTA

14. What is the surface area of a rectangular prism with dimensions 2 feet, 4 feet, and 6 feet, in feet squared?

A) 22 B) 44 C) 48 D) 88 E) NOTA

15. Given that MAO XYZ, which of the following is true?

I. $\angle M = \angle X$ II. MA = XY III. $\frac{MA}{XY} = \frac{MO}{XZ}$ IV. $\angle A = \angle Z$

A) I, III B) I, II, III C) II, IV D) I, III, IV E) NOTA

16. A rectangular pool 20 feet wide and 16 feet long has a concrete sidewalk surrounding it always 2 feet away from each side. If the concrete is negligibly thick, how much square footage of concrete covers the ground around the pool?

A) 80 B) 120 C) 160 D) 180 E) NOTA

17. What is the smaller angle (in degrees) between the hour and minute hand of an analog clock when it is 6:15?

A) 45 B) 97.5 C) 85 D) 90 E) NOTA

18. A circle is divided into 12 congruent sectors. What is the central angle in one of the sectors, to the nearest degree?

A) 25 B) 30 C) 45 D) 60 E) NOTA

19. Triangle ACB is similar to triangle DEF. If AB = 14, BC = 9, AC = 12, and DE = 8, then what is the perimeter of triangle DEF?

A) $\frac{70}{3}$ B) 35 C) 40 D) $\frac{140}{3}$ E) NOTA

20. Given right triangle ABC with right angle at A, the altitude $\overline{AD}$ is drawn to the hypotenuse such that AD = 4 and BD = 6. What is the length of segment $\overline{AC}$?

A) $\frac{3\sqrt{89}}{2}$ B) $\sqrt{119}$ C) $6\sqrt{3}$ D) $\frac{4\sqrt{13}}{3}$ E) NOTA

21. What is the hypotenuse of a triangle with leg lengths $\sqrt{63}$ and $\sqrt{65}$?

A) $\sqrt{67}$ B) $\sqrt{123}$ C) $8\sqrt{2}$ D) 8 E) NOTA

22. Consider a regular nonagon. Find the number of diagonals plus the sum of the interior angles plus the sum of the exterior angles (one at each vertex) plus the number of sides plus one. Disregard all units.

A) 1653 B) 1654 C) 1655 D) 1656 E) NOTA

23. Imagine ΔABC with $\angle C = 55°$, $\angle B = 65°$. Extend $\overline{AB}$ 3 units to a point D, so that A is between D and B; extend $\overline{AC}$ 3 units to a point E, so that A is between E and C. Find the area of triangle ADE in square units.

A) $\frac{9\sqrt{3}}{4}$ B) $9\sqrt{3}$ C) $3\sqrt{3}$ D) $\frac{7\sqrt{3}}{4}$ E) NOTA

24. If two planes intersect, then their *intersection* can be
I. a line. II. three non-collinear points. III. two intersecting lines.

A) I only B) II only C) III only D) I and II E) NOTA

25. Acute triangle ABC has two sides of length 5 and 7. Which is true of the third side of the triangle, "c"?

A) $2\sqrt{6} < c < \sqrt{74}$ B) $2 < c < \sqrt{74}$
C) $2 < c < 12$ D) $2\sqrt{6} < c < 12$ E) NOTA

26. What is the area bounded by the x-axis, the y-axis, and the line $7x + 13y = 91$, to the nearest square unit?

A) 43 B) 44 C) 45 D) 46 E) NOTA

27. What is the distance between (2, -5) and (-5, 3)?

A) $\sqrt{82}$ B) $\sqrt{97}$ C) $\sqrt{113}$ D) $\sqrt{119}$ E) NOTA

28. What is the number of sides of the first polygon to have more than 100 diagonals?

A) 15 B) 16 C) 17 D) 18 E) NOTA

29. Triangle FGH has a right angle at G. Draw right triangle GHI with right angle at H so that $\overline{IG}$ intersects $\overline{FH}$ at a point J. If FG = 4 and HI = 8, find the shortest distance between J and $\overline{GH}$, and add it to one-twelfth. Disregard units.

A) $\frac{8}{3}$ B) $\frac{11}{4}$ C) $\frac{37}{12}$ D) $4\sqrt{5}$ E) NOTA

30. How many sides does the regular polygon ABCDEFGHIJKLMN have?

A) 12 B) 13 C) 14 D) 15 E) NOTA

1. [C] It's always 180.
2. $1^2+2^2 \neq 3^2$, so [D]
3. Since the base equals the height, we have $\frac{1}{2}\bullet\frac{1}{2}\bullet\frac{1}{2}=\frac{1}{8}\,un^2$ [B]
4. $\pi r^2 = 4\pi$, so the radius is 2, and the circumference is $2\pi r = 4\pi$ [D]
5. We need to have $(2x-7)+(4x+2)<5x+9$, so $x<14$ will have the figure not being a triangle. [A]
6. We take the midpoint of side YZ, which is $\left(\frac{-2-2}{2},\frac{3-5}{2}\right)=(-2,-1)$ [C]
7. [A]
8. Let the area of them be A. So the side length of the square is $\sqrt{A}$, and the perimeter is $4\sqrt{A}$. The radius of the circle is then $\sqrt{\frac{A}{\pi}}$, of which the circumference is $2\pi\sqrt{\frac{A}{\pi}}$. The side length of the triangle is $\sqrt{\frac{4A}{\sqrt{3}}}$, so the perimeter is $3\sqrt{\frac{4A}{\sqrt{3}}}$. The question is which is largest, of which the triangle's is, about 7.89. [D]
9. It passes through the midpoint of the two points, which is (3, -1). And it is perpendicular to the y-direction, so it is x = 3. [B]
10. Since p is true, and p implies, q, then q must be true as well. [C]
11. We have a right triangle of which one leg is the distance between the chord and the diameter, which is 2. The other leg is half the length of the chord, or 5. And the length of the hypotenuse is the radius of the circle. So the radius is $\sqrt{29}$, so the area is $29\pi \approx 91$ [B]
12. The bigger square's area is 2A, so its side length is $\sqrt{2}$, and the regular square's side length is 1, so it is [A].
13. Just the first two; same side interior are supplementary: [A]
14. $2(8+24+12)=88$ [D]
15. [A] Only I and III are true.
16. If we add 4 to each dimension we have a 24 by 20 foot area, which is 480 feet squared, and we subtract the original area, which is 320, for 160. [C]
17. We have $30h-5.5m=30(6)-5.5(15)=97.5$ [B]
18. $\frac{360}{12}=30$ [B]
19. $\frac{8}{12}=\frac{EF}{9},\ EF=6$ $\quad\frac{8}{12}=\frac{DF}{14}\quad DF=\frac{28}{3}.\ 6+8+\frac{28}{3}=\frac{70}{3}$ [A]

20. AD is the geometric mean between BD and CD, so $\frac{CD}{4}=\frac{4}{6}$, so CD = 8/3.

And AC is the geometric mean between CD and BC, so $\frac{\frac{8}{3}}{AC}=\frac{AC}{\frac{26}{3}}$, so AC = $\frac{\sqrt{208}}{3}=\frac{4\sqrt{13}}{3}$ [D]

21. $\sqrt{63+65}=\sqrt{128}=8\sqrt{2}$ [C]

22. For n = 9,

$\frac{n(n-3)}{2}+180(n-2)+360+n+1=27+1260+360+9+1=1657$ [E]

23. Angle A is 60 degrees, and since that is 60, and the other two sides are the same, then all the angles are 60 degrees, and triangle ADE is an equilateral triangle. $\frac{\sqrt{3}}{4}(3^2)=\frac{9\sqrt{3}}{4}$ [A]

24. Their intersection can be nothing, a line, or another plane. I only [A]

25. The smallest side must be greater than the answer in A in order for the triangle to be acute. [A]

26. The x-intercept is x = 13, and the y-intercept is y = 7, so it is a right triangle with legs of these lengths, so the area is $\frac{1}{2}\bullet 7\bullet 13\approx 46$ [D]

27. $\sqrt{(-5-2)^2+(3--5)^2}=\sqrt{49+64}=\sqrt{113}$ [C]

28. $\frac{n(n-3)}{2}=100$. Trying the numbers, or solving using the quadratic formula, the first polygon with more than 100 diagonals is the 16-gon. [B]

29. We use the formula: $\frac{ab}{a+b}=\frac{4(8)}{4+8}=\frac{32}{12}$ + $\frac{1}{12}=\frac{33}{12}=\frac{11}{4}$ [B]

30. [C]

1. Consider a circle of radius 4 units. Let
A = the length of the diameter, in units
B = the area of the circle, in square units
C = the circumference of the circle, in units
D = 2
Give the value of A + B + C + D, to the nearest integer.

2. Consider triangle ABC with right angle at B. If angle A is 30 degrees and the length of $\overline{BC}$ is 2, let the length of $\overline{AB} = c$ and the length of $\overline{AC} = b$.
Consider also triangle XYZ with right angle at Y. If angle X is 45 degrees and the length of $\overline{XZ}$ is 2, let the length of $\overline{XY} = z$ and the length of $\overline{YZ} = x$.
To the nearest integer, give $b + c + x + z$.

3. A building in the middle of nowhere has a radar system that tracks the outside of the building. The rectangular building is 35 feet by 40 feet long. The center of the radar's scanning area is at a corner of the building. If the radar sweeps in a circular motion, and scans an area of $160\pi\ ft^2$ outside the building, then, to the nearest foot, what is the radius of the radar's circular scanning area?

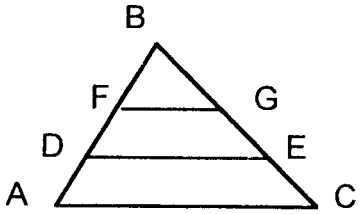
4. An equilateral triangle has two sides with lengths $2x + 5$ and $4x - 1$.
A square has two sides with lengths $y^2 + 3y + 9$ and $y^2 + 6y + 3$.
A rhombus has two sides with lengths $\frac{1}{z}$ and $\frac{2}{3}$.
Give xyz.

5. Five statements are given below, with a numerical value assigned to each statement in parentheses. Add the numbers next to the false statements, and give the sum.
(-2) If both pairs of opposite angles in a quadrilateral are congruent, then the quadrilateral is a parallelogram.
(1) You can prove that a quadrilateral is a rectangle by proving that the diagonals are congruent.
(-1) If the diagonals of a quadrilateral are perpendicular, then it is a rhombus.
(3) If a quadrilateral has all four sides congruent, then it is a square.
(-2) If a quadrilateral has both pairs of opposite sides congruent and one right angle, then it is a rectangle.

6. Let A = the area of a sector of a circle with radius 2 feet and central angle of $60°$, to the nearest square foot.
B = the straight-line distance between an 8 meter chord in a circle with diameter 10 meters, to the nearest meter.
Give A – B.

7. If an isosceles triangle has angle measures of 78 and 51, let x be the measure of the third angle, in degrees.
Let y be the number of letters in the word that is defined as the intersection of the medians of any triangle.
Let z be the number of degrees in each interior angle of a dodecagon.
Give z – x + y.

8. In triangle ABC, AB = 7 units, BC = 9 units, and AC = 10 units. Line segments $\overline{DE}$ and $\overline{FG}$ are drawn parallel to side $\overline{AC}$, with D and F on $\overline{AB}$ and E and G on $\overline{BC}$, with $\overline{DE}$ closer to $\overline{AC}$ such that the triangle is divided into three equal areas. In units, what is the length of line segment $\overline{DE}$? (Hint: If you take the ratio of areas and compare those with that of lengths, you need to take the square root of the area side of the ratio to counteract the difference in units)



9. Given triangle XYZ with right angle at Y, with the length of $\overline{XY} = 12$, the length of $\overline{YZ} = 5$. The altitude to side $\overline{XZ}$ is drawn, cutting the side of the triangle into two pieces. Give the ratio of the shorter piece to the longer piece, to the nearest hundredth.

10. A rectangular piece of cardboard has dimensions 10 inches by 12 inches. 1 inch squares are cut from each of the four corners, and the cardboard is folded into a box. Give the sum of the surface area of the box in square inches and the volume of the box in cubic inches.

11. The line with equation x + 4y = 9 intersects the x-axis at $(a, 0)$ and the y-axis at $(0, b)$. The given line also intersects another line, x + y = 0, at the point (c,d). Give the value of $a-b-c+d$, to the nearest integer.

12. Let A = the area of a square with a diagonal of length $\sqrt{2}$
B = the area of a rectangle with sides of length 4 and 8
C = the area of a parallelogram with longer side length 4 and perpendicular height to the base (the longer side) of length 4
Give ABC. Disregard all units.

13. If you need to make a pen for your dog, and you have one side of your house as one side of the pen, and you have 100 feet of fencing for the other three sides, then what is the maximum area you can enclose? Give your answer to the nearest square foot.

14. Give the area of a regular hexagon with side length 4, plus the area of a regular triangle with side length 4.

15. Triangle ABC has AB = 3, BC = 4, and AC = 5. Give the area of the triangle plus the length of the only altitude that is not a side length. Disregard units, and round to the nearest whole number.

1. A- The diameter is twice the radius, or 8.
B- The area is πr^2, or 16π
C- The circumference is $2\pi r$, or 8π
So A + B + C + D is $8+16\pi+8\pi+2 \approx \boxed{85}$

2. We have a 30-60-90 with BC = 2, so AB is $\sqrt{3}$ times that, or $2\sqrt{3}$, and the length of the hypotenuse is double, or AC = 4. The other is a 45-45-90, and since the hypotenuse is 2, the legs are both $\frac{2}{\sqrt{2}}$, or $\sqrt{2}$. $2\sqrt{3}+4+2\sqrt{2} = \boxed{10}$

3. The radar sweeps out three-quarters of a circle, so we solve $\frac{3}{4}\pi r^2 = 160\pi$, so we get approximately $r = \boxed{15}$

4. x = 3, y = 2, and z = 1.5. xyz = $\boxed{9}$

5. $1 - 1 + 3 = \boxed{3}$

6. A: $\frac{60}{360}(\pi \bullet 2^2) = \frac{2\pi}{3} \approx 2$
B: The radius is the hypotenuse of the triangle, or 5. One leg is 4, and the distance is the other leg, or 3. $2 - 3 = \boxed{-1}$

7. x can only be 78 or 51, and 78 degrees would not make it a triangle. The centroid is the intersection of the medians, so y = 8. $\frac{180(10)}{12} = 150 = z$, so z – x + y = $\boxed{107}$

8. We take the ratio of DE to AC, and we know that triangle BDE is 2/3 of the area of triangle ABC, and to counteract the difference between the units, we have to take the square root of the ratio on the right, so we have
$\frac{DE}{10} = \frac{\sqrt{2}}{\sqrt{3}}, \quad DE = \boxed{\frac{10\sqrt{6}}{3}}$

9. Let the shorter piece be "a". So $\frac{a}{5} = \frac{5}{13}$. This equation, cross-multiplied, is $a = \frac{25}{13}$. So The longer piece is $13 - \frac{25}{13} = \frac{144}{13}$. So the ratio is $\frac{25}{144} = \boxed{0.17}$

10. The area is the area of the cardboard minus the four squares, or 120 – 4 = 116. The volume is (10 – 2)(12 – 1)(1) = 80. 116 + 80 = $\boxed{196}$

11. a = 9, b = 9/4, c = -3, d = 3. $9-\frac{9}{4}+3+3\approx \boxed{13}$

12. A: So we have a 45-45-90 with hypotenuse $\sqrt{2}$, and the side length is 1, so the area is 1.
B: 4(8) = 32
C: 4(4) = 16.
ABCD = $\boxed{512}$

13. The two sides are length x, and the third side is 100 – 2x, and the area function is $100x-2x^2$, of which the maximum is at the vertex. So x = 25, and the area is (100 – 50)(25) = $\boxed{1250}$

14. $6\left(\frac{\sqrt{3}}{4}\bullet 16\right)+\left(\frac{\sqrt{3}}{4}\bullet 16\right)=\boxed{28\sqrt{3}}$

15. The area is 6, and the altitude is is 2.4, so we have $\boxed{8}$