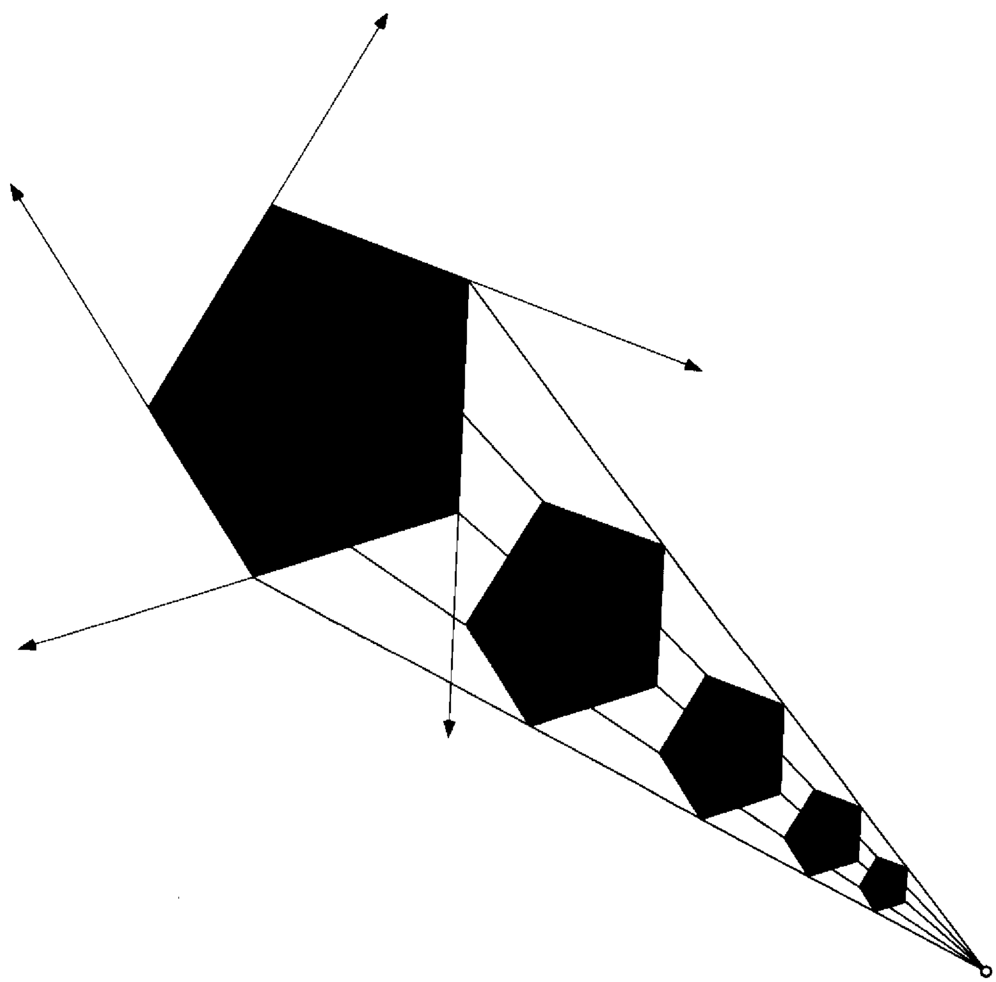
5

Polygons



Exterior Angles in a Polygon

Name(s): ____________________

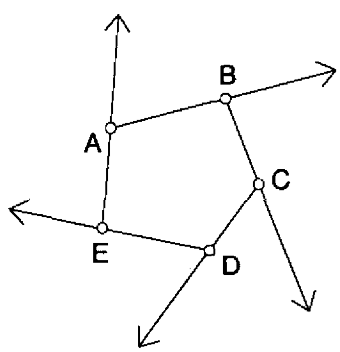
An exterior angle of a polygon is formed when one of the sides is extended. Exterior angles lie outside a convex polygon. In this investigation, you'll discover the sum of the measures of the exterior angles in a convex polygon.

Do this investigation with a triangle, a quadrilateral, or a pentagon. Plan together with classmates at nearby computers to investigate different polygons so that you can compare your results. The activity here shows a pentagon. Don't let that throw you if you're investigating a triangle or a quadrilateral—the basic steps are the same.

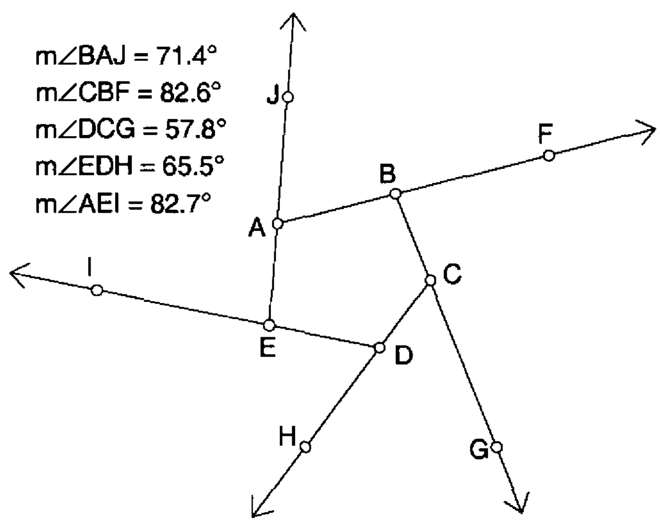
Sketch and Investigate

Hold down the mouse button on the **Segment** tool, then drag right to choose the **Ray** tool.

1. Use the **Ray** tool to construct a polygon with each side extended in one direction. Be sure to construct the polygon without creating any extra points. Your initial sketch should have the same number of points (vertices) as sides. If your polygon didn't end up convex, drag a vertex to make it convex.



Step 1



Steps 2 and 3

To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

Choose **Calculate** from the Measure menu to open the Calculator. Click once on a measurement to enter it into a calculation.

2. Construct a point on each ray outside of the polygon so that you'll be able to measure exterior angles.
3. Measure each exterior angle. Be careful to measure the correct ones!
4. Calculate the sum of the exterior angles.
5. Drag different vertices of your polygon and observe the angle measures and their sum. Be sure the polygon stays convex.
6. Compare your observations with those of classmates who did this investigation with different polygons.

Q1 Write a conjecture about the sum of the measures of the exterior angles in any polygon.

Double-click a point to mark it as a center.

Follow the steps below for another way to demonstrate this conjecture.

7. Mark any point in the sketch as a center for dilation.

In the Edit menu, choose **Select All**. Then click on each measurement to deselect it.

8. Select everything in the sketch except for the measurements.

9. Change your **Arrow** tool to the **Dilate Arrow** tool and use it to drag any part of the construction toward the marked center. Keep dragging until the polygon is nearly reduced to a single point.

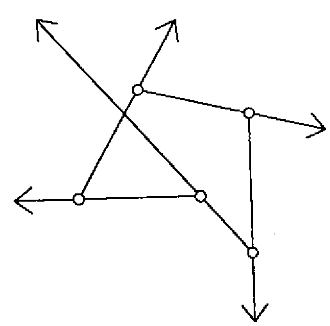
Hold down the mouse button on the **Arrow** tool, then drag right to choose the **Dilate Arrow** tool.

Q2 Write a paragraph explaining how this demonstrates the conjecture you made in Q1.

Explore More

In the Edit menu, choose **Preferences** and go to the Units panel. In the Angle Units pop-up menu, choose **directed degrees**.

1. Investigate the sum of the exterior angle measures in concave polygons. For this investigation, you may want to measure angles in directed degrees. The sign of an angle measured in directed degrees depends on the order in which you select points.

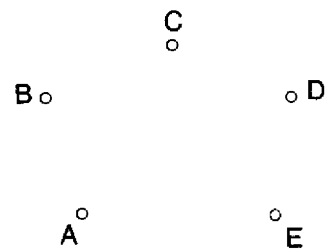


Star Polygons

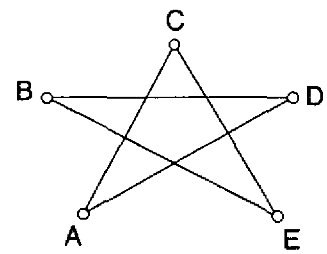
Name(s): ______________________

If you're a doodler, you probably learned at an early age how to draw a five-pointed star. In this activity, you'll discover a relationship among the angles of such a star. Then you'll investigate angles in other types of star polygons.

1. Construct five points arranged so that they roughly lie on a circle.



Step 1



Step 2

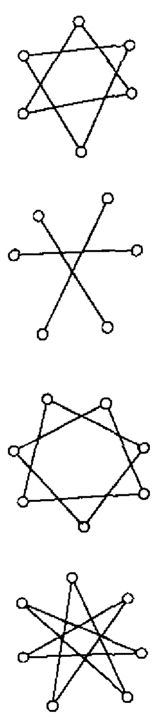
To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

2. Connect every second point with segments: $\overline{AC}$, $\overline{CE}$, $\overline{EB}$, $\overline{BD}$, and $\overline{DA}$.
3. Measure the five angles A through E at the star points.

Choose **Calculate** from the Measure menu to open the Calculator. Click once on a measurement to enter it into a calculation.

4. Calculate the sum of the five angle measures.
5. Drag any star point and observe the angle measures and the sum.

Q1 In the chart below, write the angle measure sum for the five-pointed star in which every second point is connected.



Q2 Investigate other star polygons and complete the rest of the chart. Examples of six- and seven-pointed star polygons formed by connecting every second or third point are shown at left. Plan together with classmates so that you don't have to investigate every case yourself. Also, look for patterns that you can use to fill in the chart without having to construct all the stars. Use the back of this page or a separate sheet of paper to describe any patterns you observe.

	Angle measure sums by the ways points are connected		
# of star points	Every 2nd point	Every 3rd point	Every 4th point
5			
6			
7			
8			
9			

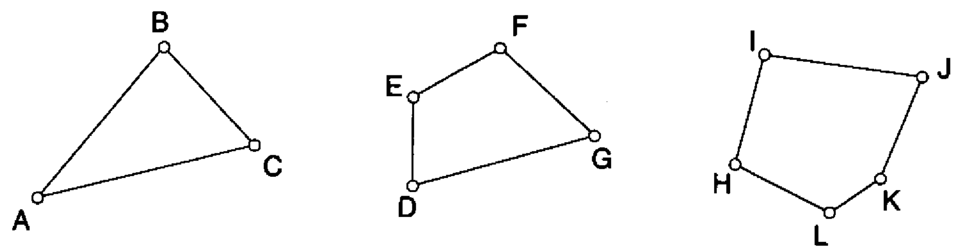
Polygon Angle Measure Sums

Name(s): ____________________

You may already know what the sum of the angle measures is in any triangle. In this activity, you'll see how that sum is related to the sum of the angle measures in other polygons.

Sketch and Investigate

1. Use the **Segment** tool to draw a triangle, a quadrilateral, and a pentagon across the top of your sketch.



To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

2. Measure each of the three angles in the triangle and arrange the measurements under the triangle.
3. Measure the four angles of the quadrilateral and arrange them under the quadrilateral.
4. Measure the five angles of the pentagon and arrange them under the pentagon.

Choose **Calculate** from the Measure menu to open the Calculator. Click once on a measurement to enter it into a calculation. If the Calculator is in the way of your measurements, move it by dragging the title bar.

5. Calculate the sum of the triangle angle measures.
6. Drag any vertex of the triangle and observe the angle measures and the sum.
7. Calculate the sum of the quadrilateral angle measures.
8. Drag any vertex of the quadrilateral and observe the angle measures and the sum. Be sure to keep the quadrilateral convex.
9. Calculate the sum of the pentagon angle measures.
10. Drag any vertex of the pentagon and observe the angle measures and the sum. Be sure to keep the pentagon convex.

Q1 Did any of the sums change when you dragged (as long as the polygons were convex)?

Q2 Describe the pattern in the angle measure sums as you increased the number of sides in a polygon.

11. Draw a diagonal in the quadrilateral.
12. Draw two diagonals from one vertex in the pentagon.

Q3 Write a paragraph explaining what these diagonals have to do with the pattern you described in Q2.

Q4 Write an expression for the sum of the angle measures in an n-gon.

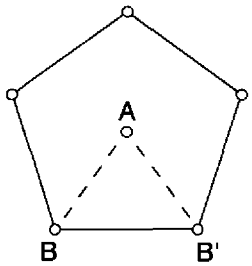
Explore More

1. Sketchpad does not display angle measures greater than 180°, so if you drag your polygon so that it's concave, Sketchpad measures the angle outside the polygon instead of the interior angle. Suppose you were able to measure angles greater than 180°. Do you think a concave polygon would have the same angle measure sum as a convex polygon with the same number of sides? Explain why or why not.

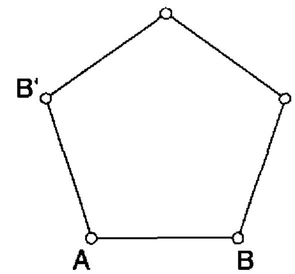
Constructing Regular Polygons

Name(s): ____________

A *regular polygon* is a polygon whose sides all have equal length and whose angles all have equal measure. The easiest way to construct regular polygons with Sketchpad is to use rotations. The figures below show pentagons constructed by two different methods.



This pentagon was constructed by rotating vertex *B* around center point *A*. ∠*BAB′* is a *central angle* of the polygon.



This pentagon was constructed by rotating vertex *B* around vertex *A*. ∠*BAB′* is an *interior angle* of the polygon.

Before you rotate anything, you must mark a center of rotation. Double-click a point to mark it as a center. Select what you want to rotate; then, in the Transform menu, choose **Rotate**.

Experiment with using rotations to construct different regular polygons. Figure out central angle measures to rotate by and interior angle measures to rotate by. Each time you make a polygon that seems correct, drag points to make sure it holds together. Make custom tools of your successful constructions to use in later work. Fill in the chart below with the central and interior angle measures for the named regular polygons, whether you have time to construct them all or not. Indicate which constructions you made custom tools for.

To make a custom tool, select the entire figure. Then, click on the **Custom** tools icon (the bottom tool in the Toolbox), and choose **Create New Tool** from the menu that appears.

Polygon	Central angle measure	Interior angle measure	Saved tool? (Y or N)
Equilateral triangle			
Square			
Regular pentagon (5)			
Regular hexagon (6)			
Regular octagon (8)			
Regular nonagon (9)			
Regular decagon (10)			
Regular *n*-gon			

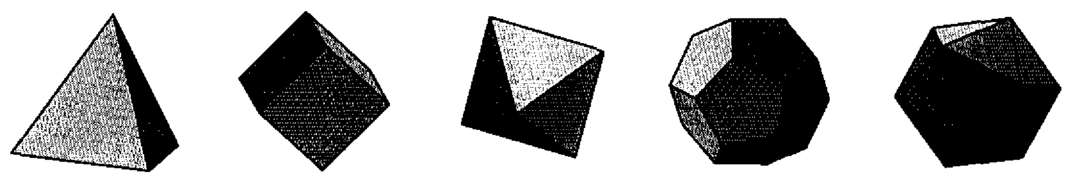
Explore More

1. The regular heptagon (seven sides) doesn't appear on the chart because the angle measures aren't "nice." What are they? To construct the regular heptagon, use the Calculator to calculate an expression for the desired angle, then mark that measurement as an angle for rotation.

Constructing Templates for the Platonic Solids

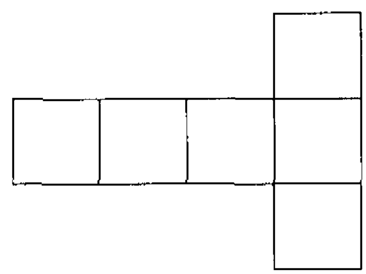
Name(s): ______________________

You've probably constructed an equilateral triangle. That construction was the first proposition in Euclid's *Elements*. After thirteen books of carefully sequenced constructions and theorems, the grand finale of Euclid's *Elements* is his proof that there are exactly five Platonic solids. A *Platonic solid* is a polyhedron whose faces are all congruent, regular polygons meeting at each vertex in the same way. The five solids are shown below.

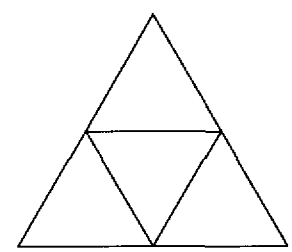


Tetrahedron | Cube | Octahedron | Dodecahedron | Icosahedron

With Sketchpad, you can construct a template for an unfolded polyhedron and print it on a piece of stiff paper. Then you can cut it out and fold it into the polyhedron, taping together the edges. Each of the solids has more than one possible template. Two examples are shown here.



Cube (Hexahedron)



Tetrahedron

As you can see, the cube has six square faces. The four faces of the tetrahedron are equilateral triangles. An octahedron has eight faces, a dodecahedron has twelve faces, and an icosahedron has twenty faces.

Use the Transform menu for reflections and rotations. You'll need to mark mirrors (segments) for reflections and to mark centers (points) for rotations.

→ Use Sketchpad to create templates for one or more of the other Platonic solids. Once you've created a regular polygon, you can use reflections and/or rotations to create an adjacent one on your template. You can also use custom tools for regular polygons to create your templates. Describe the method you used to make each template.

Before you print, in the File menu, choose **Print Preview**. Make sure the template fits on a page. If it doesn't, check the Scale To Fit Page box.

→ If you can, print your templates, cut them out, and fold them to see if they work the way you imagined.

Explore More

1. Make templates for other three-dimensional shapes: cylinders, prisms, and so on.
2. Do some research and make templates for semiregular polyhedra (called *Archimedean solids*).