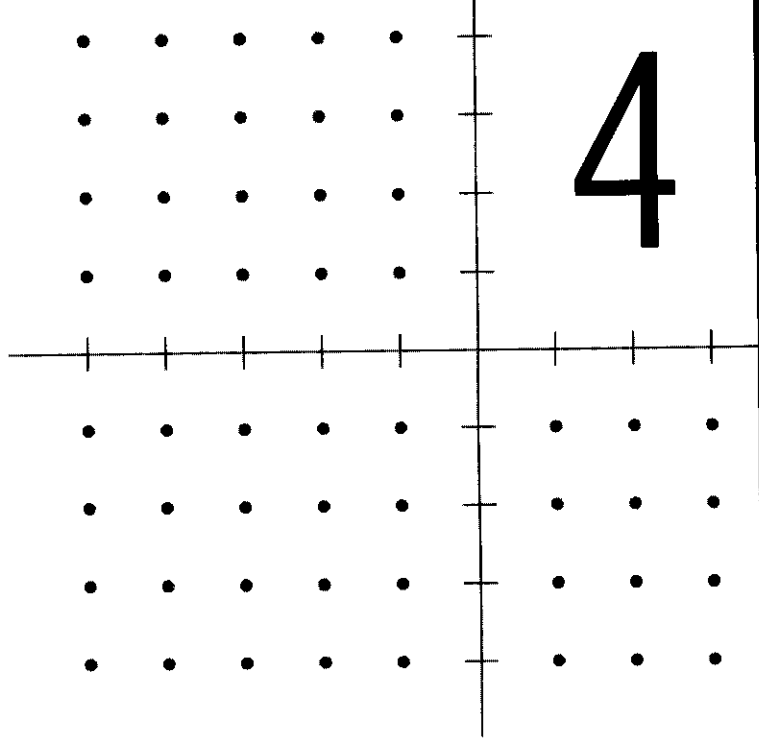
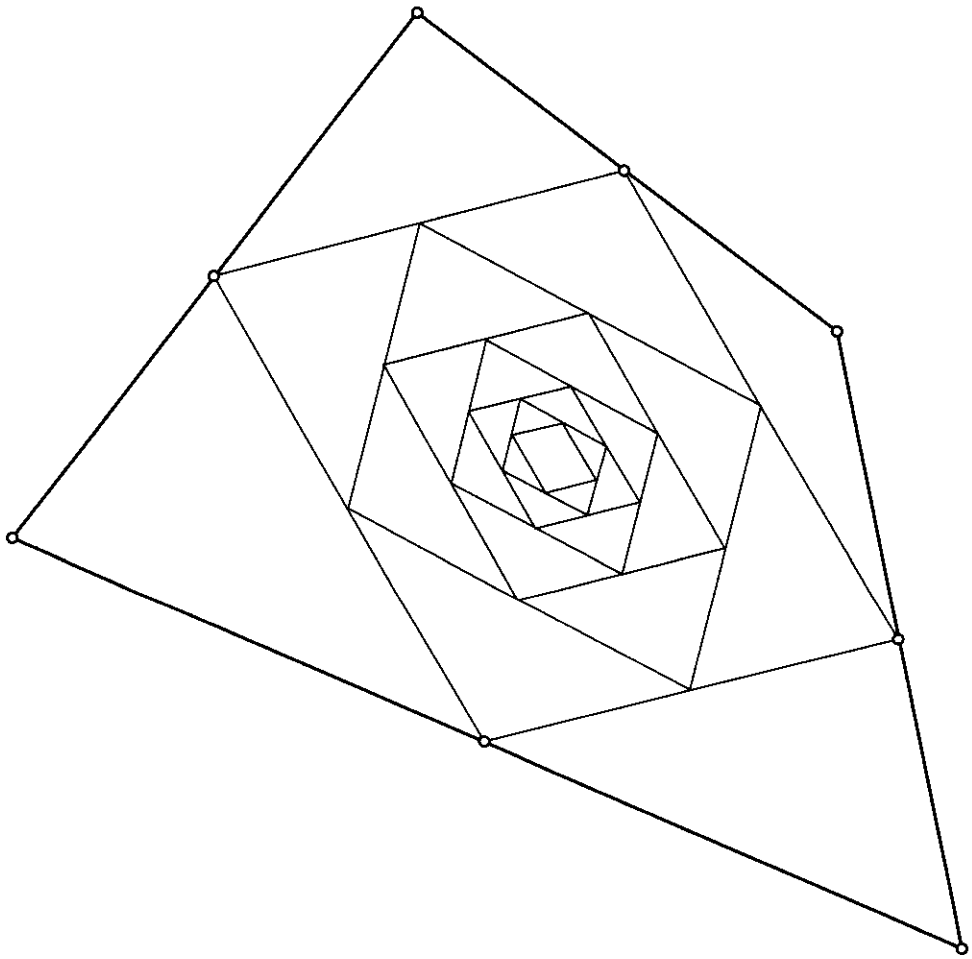


4

Quadrilaterals



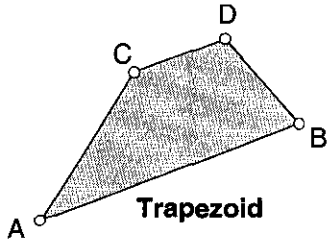
Defining Special Quadrilaterals

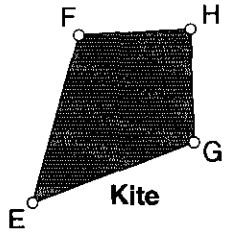
Name(s): _____

Trapezoids, kites, parallelograms, rectangles, rhombuses, and squares have special properties that distinguish them from other quadrilaterals. In this investigation, you'll experiment with these shapes to discover what makes them different from "ordinary" quadrilaterals.

Trapezoids and Kites

1. Open the document **Special Quads.gsp**.
2. Drag various parts of these quadrilaterals. Each quadrilateral has a different set of constraints in its construction that keeps it what it is.





3. Measure the slopes of the four sides of the trapezoid.

If lines (or segments) have the same slope, they are parallel.

Q1 How many pairs of sides in the trapezoid are always parallel? Use your observations to define *trapezoid*.

4. Measure the lengths of the four sides of the kite.

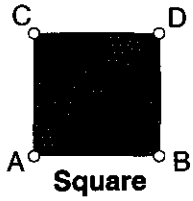
Q2 Which sides are always equal in length? Use your observations to define *kite*.

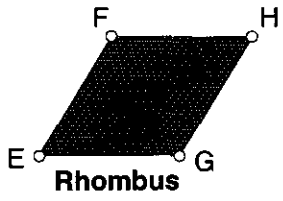
Parallelograms

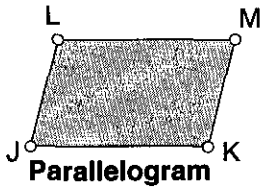
Each quadrilateral has a different set of constraints in its construction that keeps it what it is. For example, you can drag vertices of the rhombus to make different shapes, but you'll still have a rhombus.

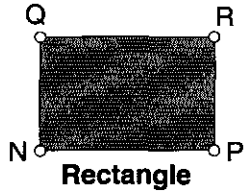
5. Go to page 2 of **Special Quads.gsp**.
6. Drag various parts of these quadrilaterals.
7. Measure the slopes of the sides of the parallelogram.

Q3 Drag parts of the parallelogram. How many pairs of sides are parallel? Use your observations to define *parallelogram*.









To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

8. Measure the angles in the rectangle.

Q4 Drag parts of the rectangle. Use your observations to define *rectangle.*

9. Measure the side lengths in the rhombus.

Q5 Drag parts of the rhombus. Use your observations to define *rhombus.*

10. Measure the side lengths and angles of the square.

Q6 Drag parts of the square. Use your observations to define *square.*

Q7 Drag the rhombus so that it's also a rectangle (or at least close to it). What's the best name for this shape? _____

Q8 Drag the rectangle so that it's also a rhombus (or at least close to it). What's the best name for this shape? _____

Q9 Based on your observations in Q7 and Q8, write a definition of *square* different from your definition in Q6.

Q10 In a–e below, circle the word—*always, sometimes,* or *never*—that makes the sentence true.

a. A parallelogram is (always/sometimes/never) a square.

b. A rectangle is (always/sometimes/never) a rhombus.

c. A square is (always/sometimes/never) a rhombus.

d. A rectangle is (always/sometimes/never) a parallelogram.

e. A parallelogram that is not a rectangle is (always/sometimes/never) a square.

Explore More

In the Display menu, choose **Show All Hidden** to get an idea of how the quadrilaterals in the **Special Quads** sketch were constructed.

1. In a new sketch, try to construct one or more of the special quadrilaterals that you defined in this activity. Make sure your quadrilateral keeps its defining properties when you drag. Describe your construction method.

Properties of Parallelograms

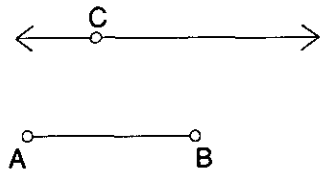
Name(s): _____

A *parallelogram* is a quadrilateral whose opposite sides are parallel. In this activity, you will construct a parallelogram using the definition, then investigate properties of parallelograms.

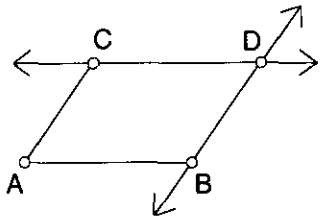
Sketch and Investigate

Select $\overline{AB}$ and point C; then, in the Construct menu, choose **Parallel Line**.

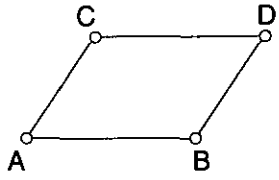
1. Construct $\overline{AB}$ and point C above the segment.
2. Construct a line through C parallel to $\overline{AB}$.



Steps 1 and 2



Steps 3–5



Step 6

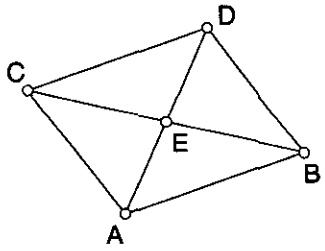
3. Construct $\overline{AC}$.
4. Construct a line through B parallel to $\overline{AC}$.
5. Construct point D, the point of intersection of the two lines.
6. Hide both lines, then finish your parallelogram by constructing the missing segments.
7. Drag different vertices of your parallelogram to make sure it's constructed properly.

To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

8. Measure the sides and angles of the parallelogram.
9. Drag different parts of the parallelogram and observe the measurements.

Q1 Write at least three conjectures about the sides and angles of a parallelogram. Use a separate sheet.

10. Construct the diagonals and their point of intersection.



Select a segment; then, in the Measure menu, choose **Length**. Or select two points and choose **Distance**.

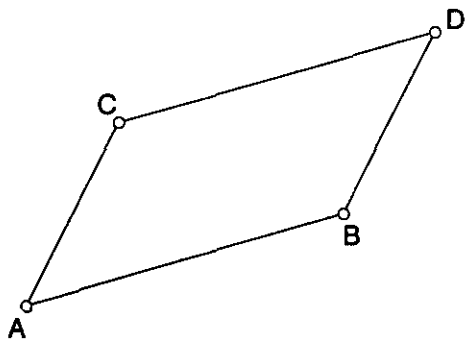
11. Drag parts of the parallelogram and observe the diagonals. Measure lengths that look as if they might be related.

Q2 Write a conjecture about the diagonals of a parallelogram. Use a separate sheet if necessary.

Constructing Parallelograms

Name(s): _____

How many ways can you come up with to construct a parallelogram? Try methods that use the Construct menu, the Transform menu, or combinations of both. Consider how you might use diagonals. Write a brief description of each construction method along with the properties of parallelograms that make that method work.



Method 1:

Properties:

Method 2:

Properties:

Method 3:

Properties:

Method 4:

Properties:

Properties of Rectangles

Name(s): _____

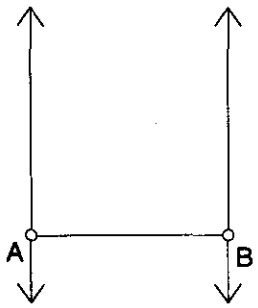
A *rectangle* is a quadrilateral with four right angles. In this investigation, you'll construct a rectangle using its definition and discover that a rectangle has many special properties besides its equal angles.

Sketch and Investigate

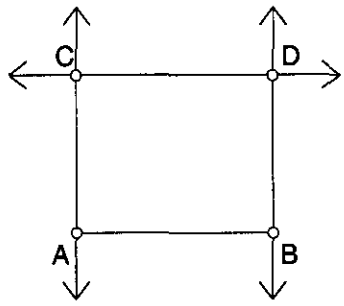
1. Construct $\overline{AB}$.

Select points *A* and *B* and $\overline{AB}$; then, in the Construct menu, choose **Perpendicular Lines**. You'll get both perpendicular lines at once.

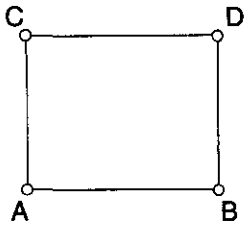
2. Construct lines perpendicular to $\overline{AB}$ through points *A* and *B*.



Steps 1 and 2



Steps 3–5

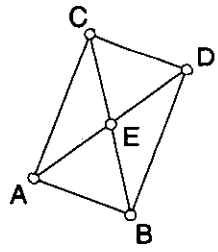


Step 6

3. Construct point *C* on the line through point *A*.
4. Construct a line through point *C* perpendicular to $\overleftrightarrow{AC}$.
5. Construct the fourth vertex, point *D*, at the intersection of this line and the line through point *B*.
6. Hide the lines, then construct segments to complete the rectangle.
7. Measure the sides of the rectangle.
8. Drag different vertices of your rectangle to make sure it's constructed properly. Observe the side lengths as you drag.

Q1 Make a conjecture about the sides of a rectangle.

9. Construct the diagonals and their point of intersection.



Select a segment; then, in the Measure menu, choose **Length**. Or select two endpoints and choose **Distance**.

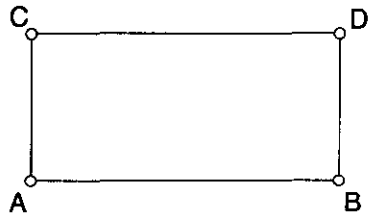
10. Drag parts of the rectangle and observe the diagonals. Measure lengths that look as if they might be related.

Q2 Write at least two conjectures about the diagonals of a rectangle. Use another sheet of paper if necessary.

Constructing Rectangles

Name(s): _____

How many ways can you come up with to construct a rectangle? Try methods that use the Construct menu, the Transform menu, or combinations of both. Consider how you might use diagonals. Write a brief description of each construction method along with the properties of rectangles that make that method work.



Method 1:

Properties:

Method 2:

Properties:

Method 3:

Properties:

Method 4:

Properties:

Properties of Rhombuses

Name(s): _____

A rhombus is an equilateral quadrilateral. In this investigation, you'll discover many other properties of rhombuses.

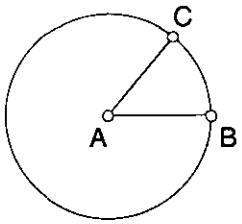
Sketch and Investigate

1. Construct circle AB.

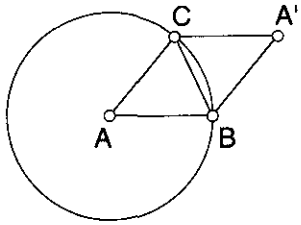
Be sure to use the circle's points as endpoints for $\overline{AB}$.

2. Construct $\overline{AB}$.

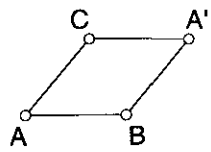
3. Construct $\overline{AC}$, where point C is a point on the circle.



Steps 1–3



Steps 4 and 5



Step 6

4. Construct $\overline{BC}$.

Double-click $\overline{BC}$ to mark it as a mirror. Select point A, $\overline{AC}$, and $\overline{AB}$; then, in the Transform menu, choose **Reflect**.

5. Mark $\overline{BC}$ as a mirror and reflect point A, $\overline{AC}$, and $\overline{AB}$ over it.

6. Hide the circle and $\overline{BC}$.

7. Drag different vertices of your rhombus to make sure it's constructed properly.

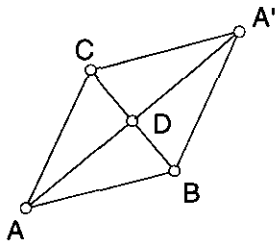
To measure slope, first select a segment. Then, in the Measure menu, choose **Slope**. To measure an angle, first select three points, with the vertex your middle selection.

8. Measure the slopes of the rhombus's sides and measure the angles.

9. Drag different vertices and observe these measures.

Q1 Write at least three conjectures about the sides and angles of a rhombus.

10. Construct the diagonals and their point of intersection.



To measure a length, select a segment. Then, in the Measure menu, choose **Length**. Or select two endpoints and choose **Distance**.

11. Drag parts of the rhombus and observe how the diagonals are related to each other and to the angles in the rhombus. Measure lengths and angles that look as if they might be related.

Q2 Write at least three conjectures about the diagonals of a rhombus. Use another sheet of paper if necessary.

Properties of Isosceles Trapezoids (continued)

Q3 Make a conjecture about angle pairs that aren't at the same base.

11. Draw the diagonals in your trapezoid.
12. Measure the lengths of the two diagonals.
13. Drag the vertices of the trapezoid and observe your measures.

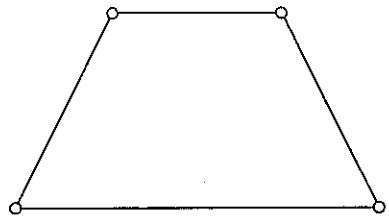
Q4 Write a conjecture about the diagonals in an isosceles trapezoid.

Explore More

1. The vertices in your construction can be dragged so that your figure is no longer a trapezoid. What shape can you make? Do your conjectures apply to this shape?
2. Investigate properties of nonisosceles trapezoids. Which, if any, of your conjectures still apply?
3. Find other ways to construct an isosceles trapezoid. Explain what you did.

Constructing Isosceles Trapezoids

Name(s): _____



How many ways can you come up with to construct an isosceles trapezoid? Try methods that use the Construct menu, the Transform menu, or combinations of both. Consider how you might use diagonals. Write a brief description of each construction method along with the properties of isosceles trapezoids that make that method work.

Method 1:

Properties:

Method 2:

Properties:

Method 3:

Properties:

Method 4:

Properties:

Midsegments of a Trapezoid and a Triangle

Name(s): _____

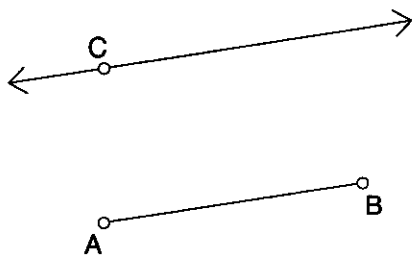
A *midsegment* in a trapezoid connects the midpoints of the two nonparallel sides. In a triangle, the midsegment connects the midpoints of any two sides. In this investigation, you'll construct a trapezoid and its midsegment and discover some properties of the midsegments of a trapezoid. Then you'll apply these properties to the special case of a triangle.

Sketch and Investigate

1. Construct $\overline{AB}$.
2. Construct point C not on $\overline{AB}$.

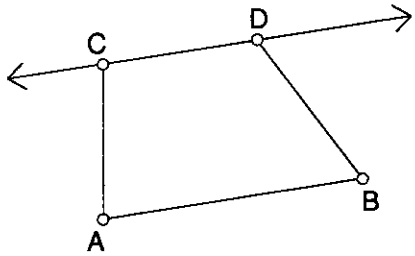
Select point C and $\overline{AB}$; then, in the Construct menu, choose **Parallel Line**.

3. Construct a line through point C parallel to $\overline{AB}$.
4. Construct $\overline{CD}$, where point D is a point on the parallel line.

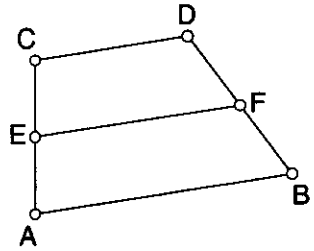


Steps 1–3

5. Construct $\overline{AC}$ and $\overline{DB}$, the legs of your trapezoid.



Steps 4 and 5



Steps 6–8

Select the line; then, in the Display menu, choose **Hide**.

6. Hide the parallel line.
7. Construct points E and F, the midpoints of $\overline{AC}$ and $\overline{DB}$.

To measure all three lengths at once, select all three segments. Then, in the Measure menu, choose **Length**.

8. Construct $\overline{EF}$, the midsegment of trapezoid $ACDB$.
9. Measure AB, EF, and CD.

Notice that EF is some number between AB and CD.

10. Drag various parts of the trapezoid and look for a relationship among the lengths of the midsegment and the bases.

Choose **Calculate** from the Measure menu to open the Calculator. Click once on a measurement to enter it into a calculation.

11. Use the measurements for AB and CD to calculate an expression equal to EF.

Q1 Write a conjecture about the midsegment of a trapezoid.

12. Measure the slopes of $\overline{AB}$, $\overline{EF}$, and $\overline{CD}$.

Q2 Write a conjecture about the slope of the midsegment of a trapezoid.

13. Drag point D until CD is as close to 0 as you can make it. Now you have a triangle.
14. Drag points A and B. Observe the relationship between AB and EF and observe the relationship between the slopes.

Q3 Make a conjecture about a midsegment of a triangle.

Explore More

1. Come up with an area formula for a trapezoid and a triangle that uses the length of a midsegment.
2. In a new sketch, draw ΔABC. Mark point B as a center and dilate points A and C by a scale factor of 1/3 (or some other scale factor). Construct $\overline{A'C'}$. How do the length and direction of $\overline{A'C'}$ compare with those of $\overline{AC}$?
3. In a new sketch, construct a triangle with all three midsegments. This divides the triangle into four smaller triangles. Investigate the properties of these triangles.
4. Create a custom tool for constructing a triangle and the midpoints of its sides. Use this tool on the midpoints of the original triangle, then on the midpoints of the newly constructed triangle, and so on. Make conjectures about the smaller successive midpoint triangles. (You can also do this exploration using **Iterate** in the Transform menu.)

Midpoint Quadrilaterals

Name(s): _____

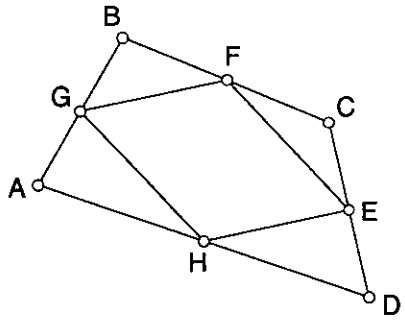
In this investigation, you'll discover something surprising about the quadrilateral formed by connecting the midpoints of another quadrilateral.

Sketch and Investigate

1. Construct quadrilateral *ABCD*.

If you select all four sides, you can construct all four midpoints at once.

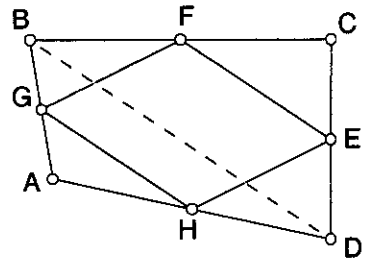
2. Construct the midpoints of the sides.
3. Connect the midpoints to construct another quadrilateral, *EFGH*.
4. Drag vertices of your original quadrilateral and observe the midpoint quadrilateral.



5. Measure the four side lengths of this midpoint quadrilateral.

Q1 Measure the slopes of the four sides of the midpoint quadrilateral. What kind of quadrilateral does the midpoint quadrilateral appear to be? How do the measurements support that conjecture?

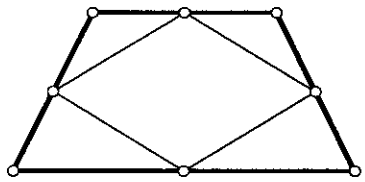
6. Construct a diagonal.
7. Measure the length and slope of the diagonal.
8. Drag vertices of the original quadrilateral and observe how the length and slope of the diagonal are related to the lengths and slopes of the sides of the midpoint quadrilateral.



Q2 The diagonal divides the original quadrilateral into two triangles. Each triangle has as a midsegment one of the sides of the midpoint quadrilateral. Use this fact and what you know about the slope and length of the diagonal to write a paragraph explaining why the conjecture you made in Q1 is true. Use a separate sheet of paper if necessary.

Special Midpoint Quadrilaterals

Name(s): _____



You may already know that if you connect the midpoints of any quadrilateral in consecutive order, you always form a parallelogram. Some quadrilaterals yield special parallelograms. For example, the midpoint quadrilateral of an isosceles trapezoid is a rhombus. What properties of an isosceles trapezoid cause its midpoint quadrilateral to be a rhombus? Are there other quadrilaterals whose midpoint quadrilaterals are always rhombuses? In this activity, you'll discover properties of the most general quadrilaterals whose midpoint quadrilaterals are special parallelograms.

Sketch and Investigate

1. Open the sketch **Special Midpoint Quads.gsp**. Drag vertices of these quadrilaterals and observe how they behave.
2. As you drag vertices of the outside quadrilaterals, observe the midpoint quadrilaterals. Decide which is always a rectangle, which is always a rhombus, and which is always a square. You should be able to tell by looking, without measuring anything.
3. Try to determine what's special about each outside quadrilateral. Add to the constructions and measure things as necessary.

Press the **Show Hint** button if you need help seeing what's special about these quadrilaterals. Once you've shown the hint, drag each quadrilateral again.

→ **Q1** Describe the properties of the following quadrilaterals:

a. A quadrilateral whose midpoint quadrilateral is a rhombus

b. A quadrilateral whose midpoint quadrilateral is a rectangle

c. A quadrilateral whose midpoint quadrilateral is a square

Explore More

1. For each of the quadrilaterals described above, explain why it has a rhombus, a rectangle, or a square for its midpoint quadrilateral.
2. In a new sketch, try each of the constructions described above yourself.

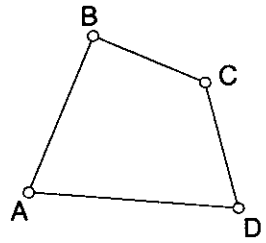
Summarizing Properties of Quadrilaterals

Name(s): _____

How much do you know about quadrilaterals? This is your chance to summarize and use what you know and maybe discover some things you didn't know. In this activity, you'll draw an arbitrary quadrilateral and measure just about everything there is to measure on it. Then you'll use those measurements to make it into various special quadrilaterals, positioning its vertices on the coordinate grid.

Preferences is in the Edit menu.

1. On the Units panel of Preferences, set Distance Units to **cm**.



Use the **Text** tool. Double-click in an empty part of your sketch to open a caption, then begin typing. You can style your text using the Text Palette that appears.

2. Construct quadrilateral *ABCD*.
3. In separate captions across the top of your sketch, type the headings Sides, Angles, Diagonals, and Slopes. (See the figure in step 11.)
4. Measure the lengths of the four sides. Arrange these measurements under the Sides heading.

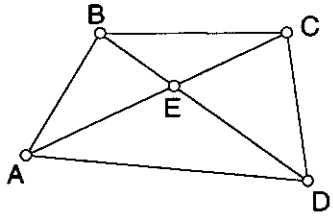
To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

5. Measure each of the four angles. Arrange these measures under the Angles heading.
6. Construct diagonals $\overline{AC}$ and $\overline{BD}$.
7. Construct point *E*, the point of intersection of the diagonals.
8. Measure the length of each diagonal and arrange these measurements under Diagonals.
9. Measure $\angle AEB$. Put this measurement under Diagonals.

Select points *A* and *E*; then, in the Measure menu, choose **Distance**. Repeat for the other distances.

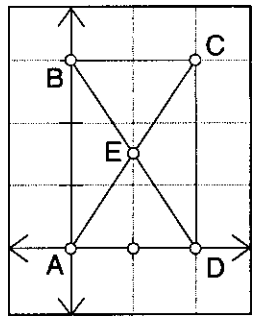
10. Measure *AE*, *EC*, *BE*, and *ED* and arrange these measurements under Diagonals.
11. Measure the slopes of the four sides. Arrange these measures under the Slopes heading.

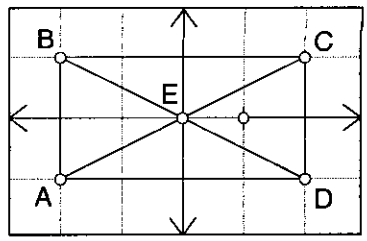
Sides	Angles	Diagonals	Slopes
m $\overline{AB}$ = 1.91 cm	m∠DAB = 65°	m $\overline{AC}$ = 3.79 cm	Slope $\overline{AB}$ = 1.677
m $\overline{BC}$ = 2.45 cm	m∠ABC = 121°	m $\overline{BD}$ = 3.37 cm	Slope $\overline{BC}$ = -0.003
m $\overline{CD}$ = 2.02 cm	m∠BCD = 98°	m∠AEB = 62°	Slope $\overline{CD}$ = -7.504
m $\overline{DA}$ = 3.71 cm	m∠CDA = 77°	AE = 2.15 cm	Slope $\overline{DA}$ = -0.099
		EC = 1.64 cm	
		BE = 1.20 cm	
		ED = 2.17 cm	



In the Graph menu, choose **Snap Points**.

12. Turn point snapping on.





13. Use all the measurements and what you know about rectangles to position the vertices on the grid in such a way that you know the quadrilateral is a rectangle. The slopes can help you: Recall that lines with equal slopes are parallel.

Take advantage of the axes when you position the vertices.

14. Try several different methods of positioning vertices so that the quadrilateral is a rectangle. Observe the measures of the sides, angles, and diagonals of these rectangles.

Q1 Fill in the "Rectangle" column in the chart below. Write "Yes" if the property is true for all rectangles and "No" if it is ever not true.

	Rectangle	Rhombus	Square	P'gram	Kite	Iso. Trap.
Opposite Sides ≅						
Opposite ∠s ≅						
Consecutive ∠'s sum = 180°						
Diagonals ≅						
Diagonals ⊥						
Diagonals bisect each other						

Positioning the rhombus is tricky. Try positioning the diagonals on the axes. Be creative about the ways you position your squares.

Q2 Experiment with positioning vertices to form parallelograms, rhombuses, squares, kites, and isosceles trapezoids. Fill in the boxes in the chart for those shapes. Remember that one counterexample is enough reason to write "No."

Q3 Which shapes named in the chart are parallelograms? How does the information in the chart support your answer? Answer on a separate sheet of paper.

Q4 A square can be defined as a rhombus and a rectangle. How does the information in the chart support this definition? Answer on a separate sheet of paper.

Q5 Is a rectangle a special kind of isosceles trapezoid? Explain.