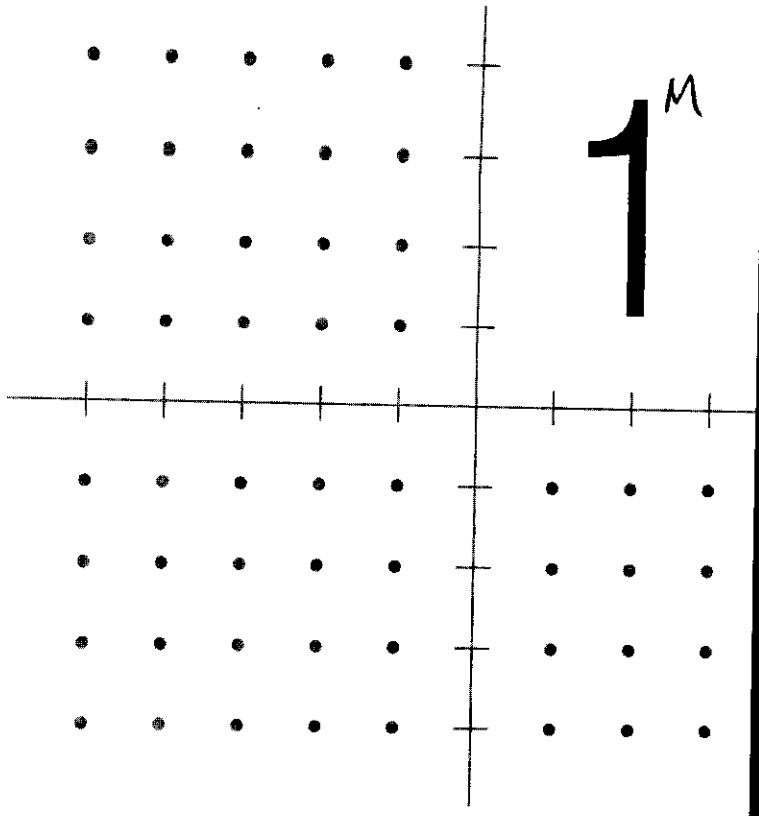

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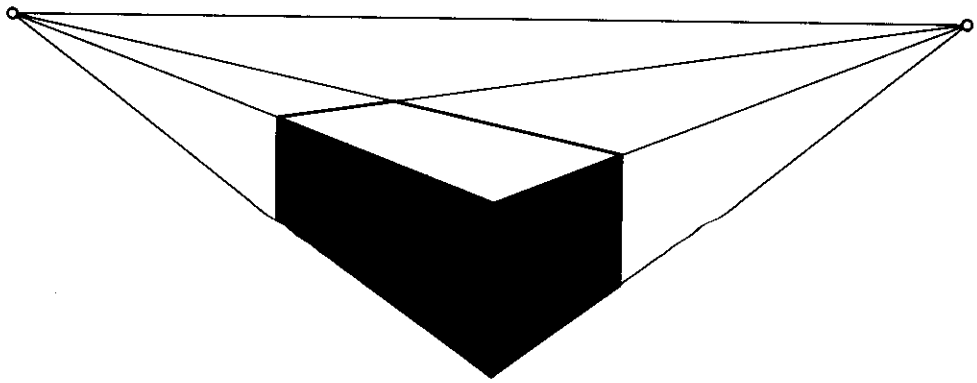
Lines and Angles



Introducing Points, Segments, Rays, and Lines

Name(s): _____

In this activity, you'll experiment with drawing, dragging, measuring, and labeling points, segments, rays, and lines. These objects, along with circles, are the building blocks of most geometric constructions.

Sketch and Investigate: Points and Segments

Note: If at any time you think you've made a mistake or you want to do something differently, you can always undo as many steps as you like. The **Undo** and **Redo** commands are in the Edit menu.


The **Point** tool

1. Choose the **Point** tool and click in the sketch to construct a point. Click again to construct a second point. Notice that the most recently constructed point is *selected*: It appears with an outline.


The **Selection Arrow** tool

2. Choose the **Selection Arrow** tool and click in a blank area in the sketch. This deselects everything.


The **Text** tool

By default, point labels start with *A*.

3. Choose the **Text** tool. Position the finger over a point, then click to display that point's label. Display the other point's label, too.

4. With the **Selection Arrow** tool, click on both points. Now both points should be selected.

5. In the Measure menu, choose **Distance**.

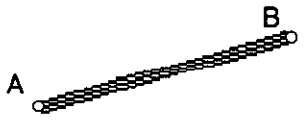
AB = 2.72 cm

6. Drag one of the points and observe the measurement.

Q1 How can you make the distance between the two points zero?


The **Segment** tool

7. Choose the **Segment** tool and draw a segment connecting the two points. You'll see a triple segment at first, indicating that the segment is selected.

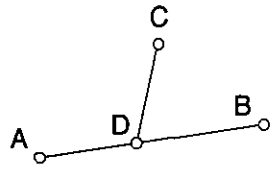


8. With the segment selected, go to the Measure menu and choose **Length**.

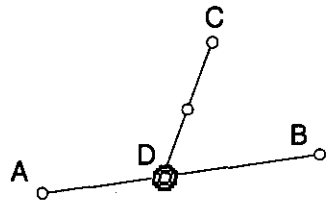
9. Use the **Selection Arrow** tool to drag either endpoint of the segment.

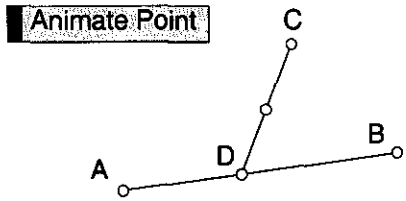
Q2 How does the length of a segment compare to the distance between its endpoints?

10. Use the **Segment** tool to construct a second segment with one endpoint attached to the first segment. To do this, click the mouse button first when the pointer is in a blank area of the sketch, then when it's directly on the original segment.



11. Use the **Text** tool to show the labels of this segment's endpoints.
12. Use the **Selection Arrow** tool to drag point D to confirm that it is attached to $\overline{AB}$.
13. Select $\overline{CD}$ (the segment, not its endpoints), then go to the Construct menu and notice what choices are available. Choose **Midpoint**.
14. Click in a blank area to deselect everything.
15. Select point D.
16. In the Edit menu, drag to the Action Buttons submenu and choose **Animation**. You'll get a dialog box you can use to specify animation settings. To choose the default settings, click OK. You've created an Animation action button in your sketch.





17. Press the action button (by clicking on it) to start the animation.
18. Press the button again to stop the animation.
19. Select the midpoint; then, in the Display menu, choose **Trace Midpoint**.
20. Press the Animation button again and observe the path that the midpoint traces.

Q3 Describe the path that the midpoint traces as point D moves back and forth.

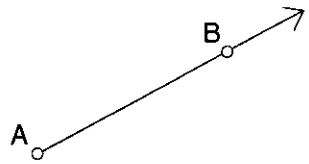
Sketch and Investigate: Rays and Lines

21. In the File menu, choose **New Sketch**.


Straightedge tool palette

22. Press and hold down the mouse button on the **Segment** tool. A palette of **Straightedge** tools will pop out to the right. Drag right and choose the **Ray** tool.

23. Draw a ray in your sketch. Notice that the ray extends in one direction beyond the edge of your sketch window.



24. Use the **Text** tool to show the labels of the ray's control points.

25. Use the **Selection Arrow** tool to drag each point to observe how it controls the ray.

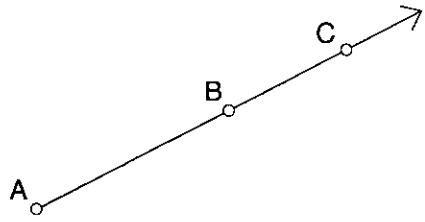
Q4 A ray with endpoint A that passes through a point B is called ray AB (represented symbolically as $\overrightarrow{AB}$). Could it also be called ray BA? Explain.

26. Select the ray and go to the Measure menu. Note that **Length** is grayed out.

Q5 Why do you think you can't measure the length of a ray?

27. With the ray still selected, go to the Construct menu and look at your choices. Choose **Point On Ray**.

Q6 Why can't you construct the midpoint of a ray?



28. Drag this new point to see how its behavior compares to that of the ray's two control points.

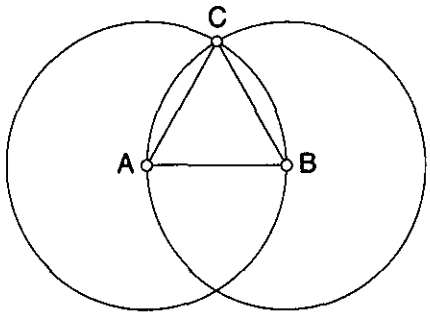
Euclid's Proposition 1— An Equilateral Triangle

Name(s): _____

Euclid, a Greek mathematician born around 300 B.C., wrote a book called the *Elements*, upon which most school geometry books are still based. All of the geometry in the *Elements* is built up sequentially from a few simple constructions and postulates. Each new property that Euclid presents, or new figure that he constructs, is based on properties he has demonstrated previously. The construction that starts it all is the equilateral triangle. Countless other constructions in the *Elements* depend on being able to construct an equilateral triangle with compass and straightedge. In this activity, you'll construct an equilateral triangle using only Sketchpad's freehand tools—the equivalents of Euclid's compass and straightedge.

Sketch and Investigate

1. Construct $\overline{AB}$.
2. Construct circle *AB*. (Make sure you use point *A* for the center and point *B* for the radius-defining point.)
3. Construct circle *BA*. (Use point *B* for the center and point *A* for the radius point this time.)



If you start or finish drawing your segment with the tip of the **Segment** tool directly over the intersection of two objects, both objects will highlight and Sketchpad will construct an endpoint at the intersection.

4. Construct $\overline{AC}$ and $\overline{CB}$, where *C* is a point of intersection of the two circles.

Q1 Drag point *A* or point *B*. What happens to your triangle? Does it appear to stay equilateral?

Q2 Explain why this triangle is always equilateral. (Hints: What roles do the circles play in your construction? How are they related to one another? How are the sides of the triangle related to the circles?)

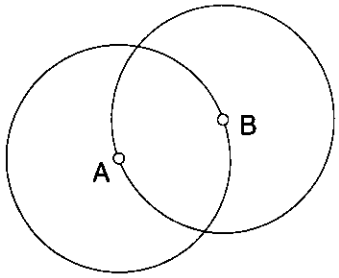
5. Hide the circles.

C

Daisy Designs

Name(s): _____

A daisy design is a simple design that you can create using only a compass. From the basic daisy, you can create more complex designs based on the regular hexagon.



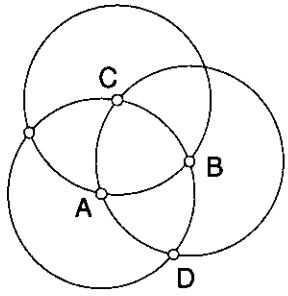
1. Construct circle *AB* (a circle with center *A* and radius point *B*).
2. Construct circle *BA*. Be sure you start your circle with the cursor positioned at point *B* and that you finish your circle with the cursor positioned at point *A*.

If both circles are not controlled by just points *A* and *B*, undo and try again. →

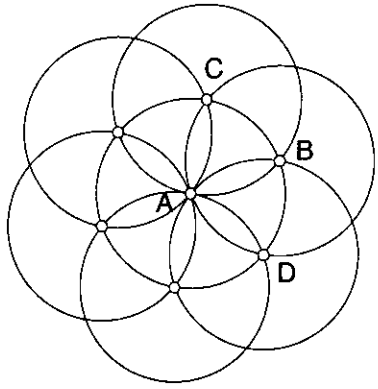
3. Drag point *A* and point *B* to confirm that both circles are controlled by these two points.
4. Construct point *C* and point *D*, the two points of intersection of these circles.

Be sure all the circles are connected by constructing them from intersections to existing points in the sketch. Your final daisy should have exactly seven points. →

5. Construct a circle from point *C* to point *A*.
6. Continue constructing circles from new intersection points to point *A*. All these circles should have equal radii. The last circle you construct should be centered at point *D*. When you're done, your sketch should look like the figure below right. You should be able to drag it without making it fall apart.



Step 5



Step 6

7. Use the **Segment** tool to add some lines to your design; then drag point *B* and observe the way your design changes.

The six points of your daisy (besides point *A*) define six vertices of a regular hexagon. You can use these points as the basis for hexagon or star designs like those shown on the next page.

You could construct polygon interiors and experiment with color. You could also construct arcs (select a circle and two points on it) and arc sector and arc segment interiors (select an arc). However, you can

Daisy Designs (continued)

probably get better results by printing out the basic line design and adding color and shading by hand. Once you have all the lines and polygon interiors you want, you can hide unneeded points. Don't hide your original two points, though, because you can use these points to manipulate your figure.

Angles Formed by Intersecting Lines (continued)

Select three angle measurements; then choose **Edit | Action Button | Hide/Show**.

5. Make a Hide/Show button for three of the four angles.
6. Press the new button (in other words, click on it). It should hide the three angle measures.

Q4 Drag a point so that the one visible angle measure is 63°. Find the measures of the other three angles without looking. Write your guess below, then press the button to check your guess.

7. Test yourself a few more times for practice: Hide the angle measures, drag to change the angles, guess at the hidden angle measures, then check your guess.

Explore More

1. Suppose you had three lines intersecting in a single point to form six angles.
   a. How many angle measures would you need to know in order to find the other angle measures?
   b. Describe any situations in which all the angles are congruent.
2. Suppose you have four lines intersecting in a single point to form eight angles. Answer parts a and b from Explore More 1, above, for this different case.
3. Now generalize your results from the last two questions. Suppose you had n lines intersecting to form $2n$ angles. Answer parts a and b from Explore More 1, above, for this general case.

Properties of Parallel Lines

Name(s): _____

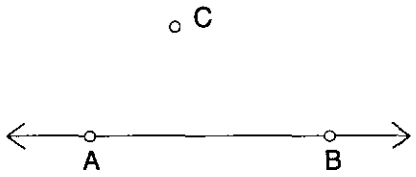
In this investigation, you'll discover relationships among the angles formed when parallel lines are intersected by a third line, called a *transversal*.

Sketch and Investigate

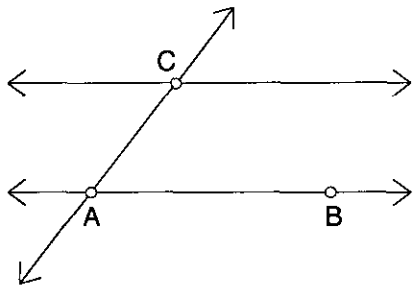
1. Construct $\overleftrightarrow{AB}$ and point C, not on $\overleftrightarrow{AB}$.

Select the line and the point; then, in the Construct menu, choose **Parallel Line**.

2. Construct a line parallel to $\overleftrightarrow{AB}$ through point C.
3. Construct $\overleftrightarrow{CA}$. Drag points C and A to make sure the three lines are attached at those points.



Step 1



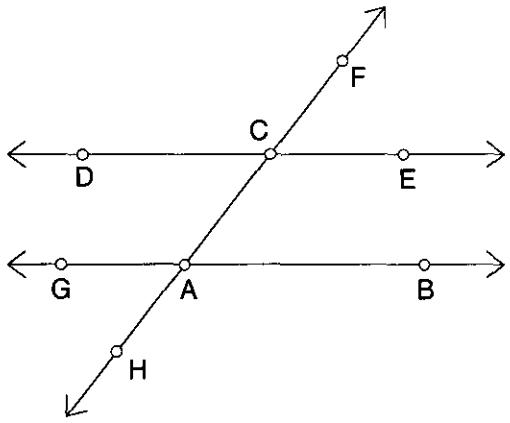
Steps 2 and 3

Using the **Text** tool, click once on a point to show its label. Double-click the label to change it.

4. Construct points D, E, F, G, and H as shown at right.

To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

5. Measure the eight angles in your figure. Be systematic about your measuring to be sure you don't measure the same angle twice.
6. Drag point A or B and see which angles stay congruent. Also drag the transversal $\overleftrightarrow{CA}$. (Be careful not to change the point order on your lines. That would change some angles into other angles.) Observe how many of the eight angles you measured appear to be always congruent.



Step 4

Q1 When two parallel lines are crossed by a transversal, the pairs of angles formed have specific names and properties. The chart on the next page shows one example of each type of angle pair. Fill in the chart with a second angle pair of each type, then state what relationship, if any, you observe between the angles in a pair type.

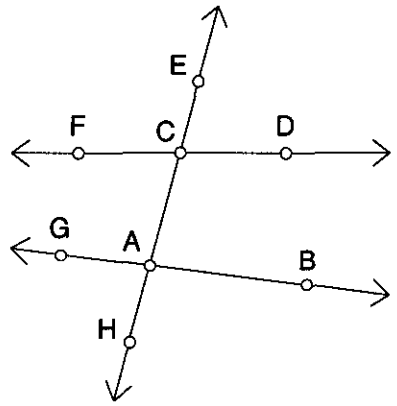
Properties of Parallel Lines (continued)

Angle Type	Pair 1	Pair 2	Relationship
Corresponding	$\angle FCE$ and $\angle CAB$		
Alternate interior	$\angle ECA$ and $\angle CAG$		
Alternate exterior	$\angle FCE$ and $\angle HAG$		
Same-side interior	$\angle ECA$ and $\angle BAC$		
Same-side exterior	$\angle FCD$ and $\angle HAG$		

Q2 One of the angle types has more than one pair. Name that angle type in the chart below, and name the third and fourth pairs of angles of that type.

Angle Type	Pair 3	Pair 4	Relationship

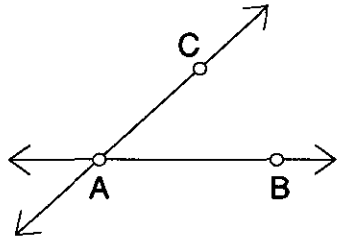
7. Next, you'll investigate the converses of your conjectures. In a new sketch, draw two lines that are not quite parallel. Construct a transversal.
8. Add points as needed, then measure all eight angles formed by the three lines.
9. Move the lines until you have two sets of four congruent angles.



Q3 If two lines are crossed by a transversal so that corresponding angles, alternate interior angles, and alternate exterior angles are congruent, what can you say about the lines?

Explore More

1. You can use the converse of the parallel-lines conjecture to construct parallel lines. Construct a pair of intersecting lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ as shown. Select, in order, points C, A, and B. Then, in the Transform menu, choose **Mark Angle**. Double-click point C to mark it as a center for rotation. You figure out the rest. Explain why this method works.

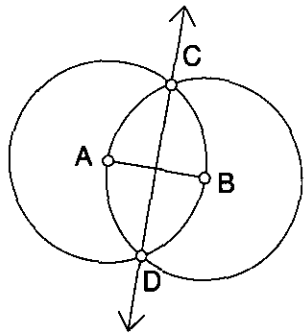


Constructing a Perpendicular Bisector

Name(s): _____

In this activity, you'll use only Sketchpad's freehand tools to construct perpendicular bisectors. Then you'll investigate properties of perpendicular bisectors. In Explore More, you'll devise a shortcut for constructing a perpendicular bisector using Sketchpad's Construct menu.

1. Construct $\overline{AB}$.
2. Construct circle AB. (Make sure you use point A for the center and point B for the radius endpoint.)
3. Construct circle BA. (Use point B for the center and point A for the radius point.)



Press and hold down the mouse button on the current **Straightedge** tool, then drag to choose the **Line** tool.

4. Construct $\overleftrightarrow{CD}$, where C and D are the circles' points of intersection.
5. Drag points A and B to make sure your construction stays together.

Q1 Line CD is the perpendicular bisector of $\overline{AB}$. Without measuring, what can you say about the distances AC and BC and the distances AD and BD?

6. Construct E, the point of intersection of $\overline{AB}$ and $\overleftrightarrow{CD}$.

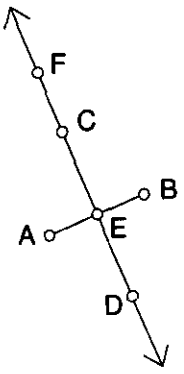
Q2 What's special about point E? Move points A and B to confirm your answer.

7. Hide the circles.
8. Construct a point F on $\overleftrightarrow{CD}$.

To measure a distance, select two points; then, in the Measure menu, choose **Distance**.

9. Measure the distances FA and FB.

Q3 Drag point F up and down the line. Make a conjecture about any point on a segment's perpendicular bisector.



Explore More

For tips on making and using custom tools, choose **Toolbox** from the Help menu, then click on the Custom Tools link.

1. In a new sketch, construct a segment. Figure out how to construct the perpendicular bisector of the segment using the Construct menu. When you've succeeded, make a custom tool and save it in the Tool Folder (next to the application itself on your hard drive). Write a description of the way you did the construction.

Drawing a Box with Two-Point Perspective

Name(s): _____

Even though a piece of paper or a computer screen is flat, you can draw figures that appear three-dimensional on these two-dimensional surfaces by using perspective drawing. In three dimensions, objects that are farther away appear smaller. Perspective drawing takes advantage of this principle to make flat drawings appear to have depth.

Follow these steps to draw a box with two-point perspective. Labels are shown to clarify these directions, but you probably won't want labels on your drawing.

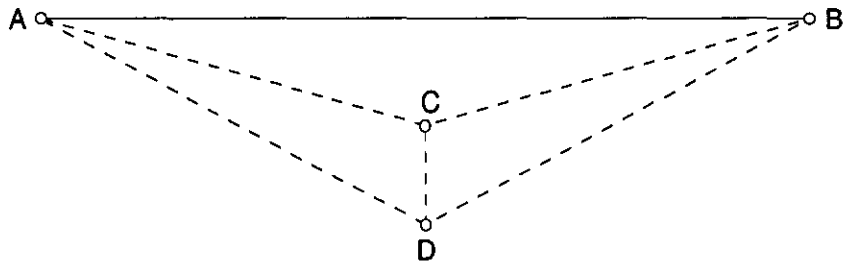
It's easier to draw a horizontal or vertical line if you hold down the Shift key while you draw.

1. Draw a long horizontal line segment AB. This will be your *horizon line*, and its endpoints will be the *vanishing points* of your perspective box.

Line Width is in the Display menu.

2. Draw a short vertical segment CD below your horizon line. This will be the front edge of your box. Change its line width to dashed.

3. Construct $\overline{CA}$, $\overline{DA}$, $\overline{CB}$, and $\overline{DB}$.



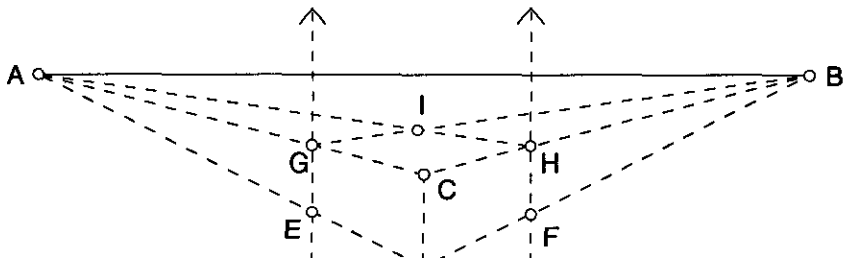
4. Construct point E on $\overline{DA}$ and point F on $\overline{DB}$.

Select $\overline{CD}$ and points E and F; then, in the Construct menu, choose **Parallel Lines**. You should get both parallel lines at once.

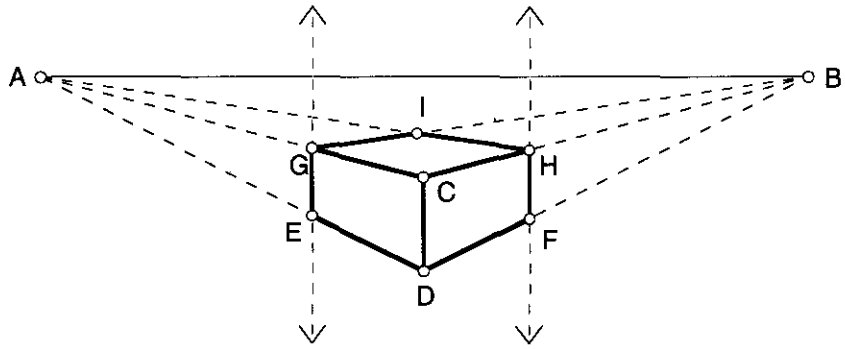
5. Through points E and F, construct lines parallel to $\overline{CD}$.

6. Construct $\overline{GB}$ and $\overline{HA}$, where points G and H are the points where the parallel lines intersect $\overline{CA}$ and $\overline{CB}$.

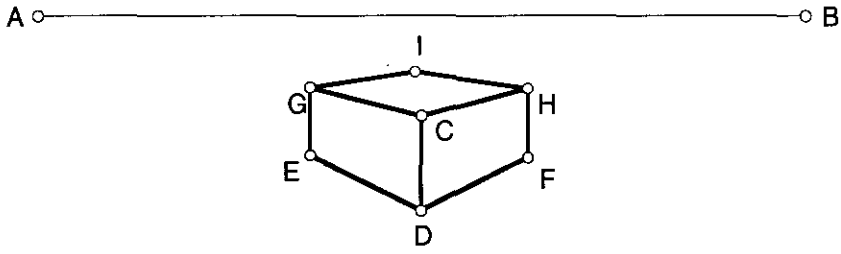
7. Construct point I at the point of intersection of $\overline{GB}$ and $\overline{HA}$.



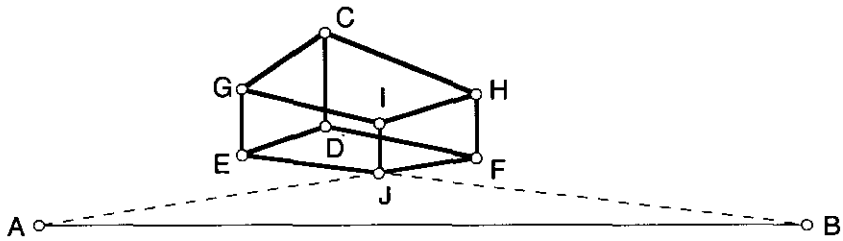
8. Change the line width of $\overline{CD}$ to thick, then complete the box by constructing $\overline{GI}$, $\overline{IH}$, $\overline{HF}$, $\overline{FD}$, $\overline{DE}$, $\overline{EG}$, $\overline{GC}$, and $\overline{CH}$.



9. Hide all the dashed lines and segments to leave just your box and the horizon line.



10. Try moving various parts of your box, your horizon segment, and the vanishing points. If you move the front edge of your box above the horizon segment, you'll discover that you haven't created the bottom of your box. Continue sketching to construct the missing edges as shown. (*Hint:* Start by constructing a dashed segment from point E to point B.)



11. Hide unwanted points and segments.