1975 - BC 4

4. (a) Determine whether the series

$$\frac{1}{3} - \frac{2^3}{3^2} + \frac{3^3}{3^3} - \frac{4^3}{3^4} + \ldots + \frac{(-1)^{n-1}n^3}{3^n} + \ldots$$

is convergent. Justify your answer.

(b) Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$. Justify your answer.

1976 - BC 7

7. (a) Write the first three nonzero terms and the general term of the Taylor series expansion about $x = 0$ of $f(x) = 5 \sin \frac{x}{2}$.

(b) What is the interval of convergence for the series found in (a)? Show your method.

(c) What is the minimum number of terms of the series in (a) that are necessary to approximate $f(x)$ on the interval $(-2, 2)$ with an error not exceeding 0.1? Show your method.

1977 - BC 5

5. (a) Does the series $\sum_{j=1}^{\infty} \frac{2}{j^2} \sin\left(\frac{\pi}{j}\right)$ converge? Justify your answer.

(b) Express $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{2}{n} \sin\left(\frac{k\pi}{n}\right)$ as a definite integral and evaluate the integral.

1978 - BC5

5. The power series $\sum_{n=0}^{\infty} \frac{\ln(n+1)}{n+1} x^n$ has the interval of convergence $-1 \leq x < 1$. Let $f(x)$ be its sum.

(a) Find $f(0)$ and $f'(0)$.

(b) Justify that the interval of convergence is $-1 \leq x < 1$.

1979 - BC 4

4. Let f be the function defined by $f(x) = \frac{1}{1 - 2x}$.

(a) Write the first four terms and the general term of the Taylor series expansion of $f(x)$ about $x = 0$.

(b) What is the interval of convergence for the series found in part (a)? Show your method.

(c) Find the value of f at $x = -\frac{1}{4}$. How many terms of the series are adequate for approximating $f\left(-\frac{1}{4}\right)$ with an error not exceeding one per cent? Justify your answer.

1980 - BC3

3. (a) Determine whether the series $A = \sum_{n=1}^{\infty} \frac{4n}{n^2 + 1}$ converges or diverges. Justify your answer.

(b) If S is the series formed by multiplying the nth term in A by the nth term in $\sum_{n=1}^{\infty} \frac{1}{2n}$, write an expression using summation notation for S.

(c) Determine whether the series S found in part (b) converges or diverges. Justify your answer.

1981 – BC 3

3. Let S be the series $S = \sum_{n=0}^{\infty} \left(\frac{t}{1+t}\right)^n$ where $t \neq 0$.

   (a) Find the value to which S converges when $t = 1$.

   (b) Determine the values of t for which S converges. Justify your answer.

   (c) Find all values of t that make the sum of the series S greater than 10.

1982 - BC 5

5. (a) Write the Taylor series expansion about $x = 0$ for $f(x) = \ln(1 + x)$. Include an expression for the general term.

(b) For what values of x does the series in part (a) converge?

(c) Estimate the error in evaluating $\ln\left(\frac{3}{2}\right)$ by using only the first five nonzero terms of the series in part (a). Justify your answer.

(d) Use the result found in part (a) to determine the logarithmic function whose Taylor series is

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{2n}}{2n}.$$

1983 - BC 5

5. Consider the power series $\sum_{n=0}^{\infty} a_n x^n$, where $a_0 = 1$ and $a_n = \left(\frac{7}{n}\right) a_{n-1}$ for $n \geq 1$.

(a) Find the first four terms and the general term of the series.

(b) For what values of x does the series converge?

(c) If $f(x) = \sum_{n=0}^{\infty} a_n x^n$, find the value of $f'(1)$.

1984 - BC 4

4. Let f be the function defined by $f(x) = \sum_{n=1}^{\infty} \frac{x^n n^n}{3^n n!}$ for all values of x for which the series converges.

(a) Find the radius of convergence of this series.

(b) Use the first three terms of this series to find an approximation of $f(-1)$.

(c) Estimate the amount of error involved in the approximation in part (b). Justify your answer.

1986 - BC5

5. (a) Find the first four nonzero terms in the Taylor series expansion about $x = 0$ for $f(x) = \sqrt{1+x}$.

(b) Use the results found in part (a) to find the first four nonzero terms in the Taylor series expansion about $x = 0$ for $g(x) = \sqrt{1+x^3}$.

(c) Find the first four nonzero terms in the Taylor series expansion about $x = 0$ for the function h such that $h'(x) = \sqrt{1+x^3}$ and $h(0) = 4$.

1987-BC4

4. (a) Find the first five terms in the Taylor series about $x = 0$ for $f(x) = \frac{1}{1 - 2x}$.

(b) Find the interval of convergence for the series in part (a).

(c) Use partial fractions and the result from part (a) to find the first five terms in the Taylor series about $x = 0$ for $g(x) = \frac{1}{(1 - 2x)(1 - x)}$.