2011 Math Olympics

Duration: 90 minutes
Neither calculators not formula sheets are permitted at any time while taking the exam.

Student name:_____

School name: _____

Solve.

1) $|2x + 7| < 19$ 1) _____

A) $(-\infty, 2)$ B) $(-13, 6)$
C) $(-\infty, -13)$ D) $(-\infty, -13) \cup (6, \infty)$

2) How many 2-letter codes can be formed with the letters P, Q, R, S, T without repetition? 2) _____

A) 4 B) 40 C) 10 D) 20

3) $\frac{8x}{x-8} - \frac{4}{x} = \frac{32}{x^2 - 8x}$ 3) _____

A) $\frac{1}{4}, -\frac{1}{4}$ B) $\frac{1}{2}, -\frac{1}{2}$ C) 2 D) $\frac{1}{2}$

4) When a single card is drawn from an ordinary 52-card deck, find the probability of getting a jack. 4) _____

A) $\frac{1}{52}$ B) $\frac{1}{13}$ C) $\frac{1}{26}$ D) $\frac{1}{4}$

5) $\frac{x+8}{x+1} < 9$ 5) _____

A) $(-1, -\frac{1}{8})$ B) $\emptyset$

C) $(-\infty, -1) \cup \left(-\frac{1}{8}, \infty\right)$ D) $(-\infty, -\frac{1}{8}) \cup (1, \infty)$

6) How many lines are determined by 18 points, no 3 of them being collinear? 6) _____

A) 18 B) 2448 C) 153 D) 306

Solve the problem.

7) A company makes 3 types of cable. Cable A requires 3 black, 3 white, and 2 red wires. B requires 1 black, 2 white, and 1 red. C requires 2 black, 1 white, and 2 red. They used 95 black, 100 white and 80 red wires. How many of each cable were made? 7) _____

A) 15 cable A
16 cable B
15 cable C

B) 48 cable A
20 cable B
12 cable C

C) 15 cable A
20 cable B
15 cable C

D) 20 cable A
15 cable B
15 cable C

Factor and simplify.

8) $\sec^4 x - 2 \sec^2 x \tan^2 x + \tan^4 x$ 8) _____

A) $\sec^2 x + \tan^2 x$

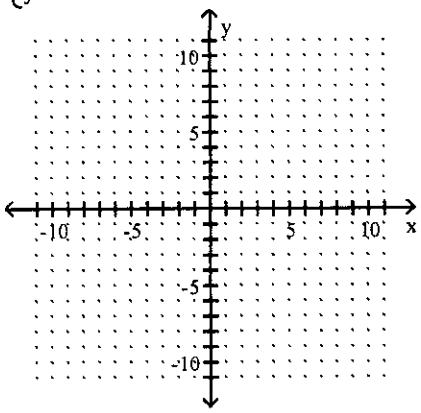
B) 2

C) 1

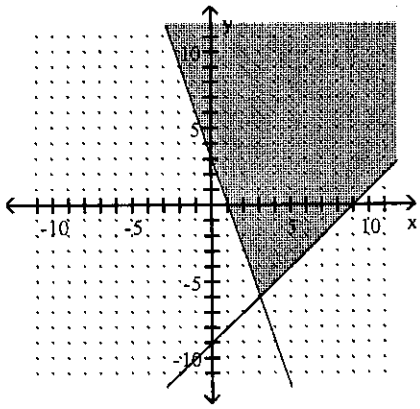
D) $\sec^2 x (1 + \tan^2 x)$

Graph the solution of the system of linear inequalities.

9) $\begin{cases} y \le -3x + 3 \\ y \ge x - 9 \end{cases}$ 9) _____



A)



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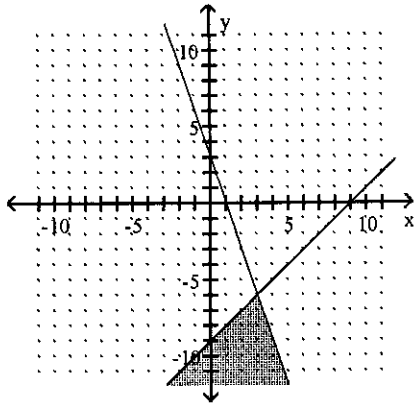

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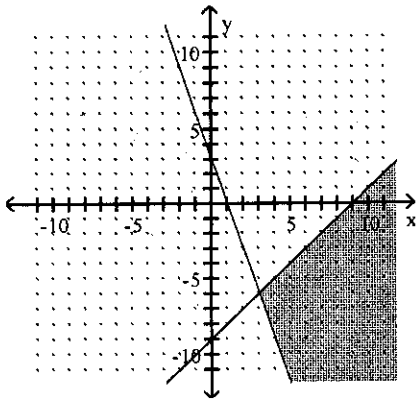

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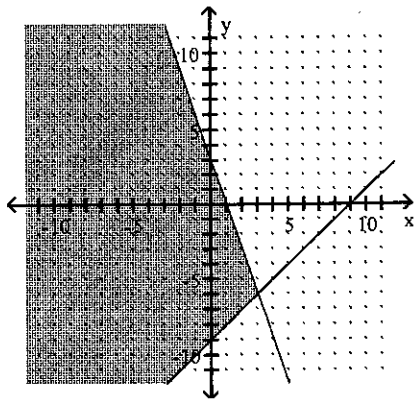


Find the trigonometric function value of angle θ.

10) $\sin\theta = -\frac{3}{11}$ and θ in quadrant III 10) _____

Find sec θ.

A) $\frac{\sqrt{11}}{3}$ B) $-\frac{3\sqrt{7}}{28}$ C) $-\frac{11\sqrt{7}}{11}$ D) $-\frac{11\sqrt{7}}{28}$

Solve the problem.

11) John does 14 pushups on the first day of a 30-day month, and then increases the number of pushups by 2 pushups a day. How many pushups has he done by the end of the month? 11) _____

A) 1290 pushups B) 1276 pushups C) 1320 pushups D) 1350 pushups

Solve, finding all solutions in [0, 2π).

12) $2\sin x\cos x - 2\sin x + \cos x = 1$ 12) _____

A) $0, \frac{5\pi}{6}, \frac{7\pi}{6}$ B) No solution C) $0, \frac{5\pi}{6}, \frac{11\pi}{6}$ D) $0, \frac{7\pi}{6}, \frac{11\pi}{6}$

13) $\sin 2x + \sin x = 0$ 13) _____

A) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ B) $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ C) $\frac{\pi}{8}, \frac{9\pi}{8}$ D) No solution

14) $4\cos x + 4\sin x = \sqrt{24}$ 14) _____

A) $\frac{\pi}{12}, \frac{5\pi}{12}$ B) $\frac{2\pi}{3}, \frac{5\pi}{6}$ C) $\frac{\pi}{6}, \frac{5\pi}{6}$ D) $\frac{\pi}{12}, \frac{7\pi}{12}$

Find the center and the radius of the circle.

15) $y^2 + 14y + 49 = 10x - x^2 - 9$ 15) _____

A) (4, 7); r = 6 B) (5, 8); r = 5 C) (5, 9); r = 7 D) (5, −7); r = 4

Find the exact value of the expression using the provided information.

16) Find $\cos(\theta + \phi)$ given that $\cos\theta = \frac{5}{13}$ and $\cos\phi = \frac{4}{5}$ and that θ and φ are between 0 and π/2. 16) _____

A) $-\frac{33}{65}$ B) $-\frac{16}{65}$ C) $\frac{63}{65}$ D) $\frac{56}{65}$

Evaluate.

17) $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)$ 17) _____

A) $\frac{4\pi}{3}$ B) $-\frac{\pi}{3}$ C) $-\frac{\pi}{6}$ D) $\frac{5\pi}{6}$

Find the sum, if it exists.

18) $\sum_{i=1}^{\infty} 29\left(\frac{5}{8}\right)^{i-1}$ 18) _____

A) $\frac{8}{3}$ B) $\frac{145}{3}$ C) $\frac{232}{3}$ D) Does not exist

19) $-15 - \frac{10}{3} - \frac{20}{27} - \frac{40}{243} - \ldots$ 19) _____

A) $-\frac{135}{7}$ B) $\frac{9}{7}$ C) $-\frac{30}{7}$ D) Does not exist

Find the exact value in radians.

20) $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$ 20) _____

A) $-\frac{\pi}{3}$ B) $\frac{2\pi}{3}$ C) $\frac{5\pi}{6}$ D) $-\frac{\pi}{6}$

Find.

21) $\tan\left(\sin^{-1}\frac{7}{y}\right)$ 21) _____

A) $\frac{7}{\sqrt{y^2 - 49}}$ B) $\frac{7}{y^2 + 49}$ C) $\frac{7}{\sqrt{y^2 + 49}}$ D) $\frac{y^2 + 49}{7}$

Express as a single logarithm and, if possible, simplify.

22) $\log_a \frac{3}{\sqrt{x}} - \log_a \sqrt{3x}$ 22) _____

A) $\log_a \frac{\sqrt{3x}}{6}$ B) $\log_a \left(\frac{3}{\sqrt{x}} - \sqrt{3x}\right)$

C) $\log_a \frac{\sqrt{3}}{x}$ D) $\log_a \frac{1}{x}$

Find the exact value.

23) Given that $\sin\theta = -\frac{4}{5}$ with θ in quadrant IV, find $\sin 2\theta$. 23) _____

A) $\frac{24}{25}$ B) $-\frac{7}{25}$ C) $\frac{7}{25}$ D) $-\frac{24}{25}$

Determine whether the given function is one-to-one. If it is one-to-one, find a formula for the inverse.

24) $f(x) = \frac{-4x - 5}{-2x - 3}$ 24) _____

A) $f^{-1}(x) = \frac{3x - 5}{-2x + 4}$ B) $f^{-1}(x) = \frac{-2x + 4}{3x - 5}$

C) Not one-to-one D) $f^{-1}(x) = \frac{-4x - 5}{-2x - 3}$

Answer the question.

25) How can the graph of $f(x) = \frac{1}{2}(x + 6)^2 - 5$ be obtained from the graph of $y = x^2$? 25) _____

A) Shift it horizontally 6 units to the right. Stretch it vertically by a factor of 2. Shift it 5 units up.

B) Shift it horizontally 6 units to the left. Shrink it vertically by a factor of $\frac{1}{2}$. Shift it 5 units down.

C) Shift it horizontally 6 units to the right. Shrink it vertically by a factor of $\frac{1}{2}$. Shift it 5 units down.

D) Shift it horizontally 6 units to the left. Shrink it vertically by a factor of 2. Shift it 5 units down.

For the pair of functions, find the indicated composition.

26) $f(x) = 4x^2 + 3x + 5$, $g(x) = 3x - 4$ 26) _____

Find $(g \circ f)(x)$.

A) $12x^2 + 9x + 19$ B) $12x^2 + 9x + 11$ C) $4x^2 + 9x + 11$ D) $4x^2 + 3x + 1$

For the pair of functions, find the indicated domain.

27) $f(x) = x^2 - 1$, $g(x) = 2x + 3$ 27) _____

Find the domain of g/f.

A) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

B) $\left[-\frac{3}{2}, \infty\right)$

C) $(-\infty, \infty)$

D) $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

Find the exact trigonometric function value.

28) $\sin(-240°)$ 28) _____

A) $\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{1}{2}$ D) -1

29) $\csc(-2040°)$ 29) _____

A) -2 B) $\frac{2\sqrt{3}}{3}$ C) $-\frac{\sqrt{3}}{3}$ D) $\sqrt{2}$

Given the polynomial function f(x), find the rational zeros, then the other zeros (that is, solve the equation $f(x) = 0$), and factor f(x) into linear factors.

30) $f(x) = x^3 + 4x^2 - 2x - 8$ 30) _____

A) $-4, -2, 2$; $f(x) = (x + 4)(x + 2)(x - 2)$

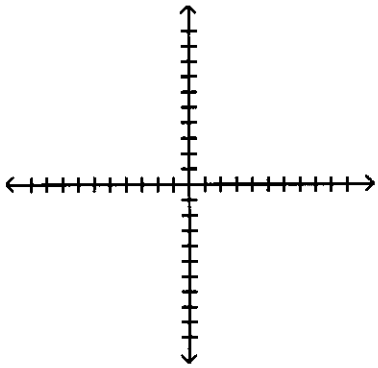
B) $-4, -\sqrt{2}, \sqrt{2}$; $f(x) = (x + 4)(x + \sqrt{2})(x - \sqrt{2})$

C) $-\sqrt{4}$, multiplicity 2; $-\sqrt{2}$, multiplicity 2; $f(x) = (x + \sqrt{4})^2(x + \sqrt{2})^2$

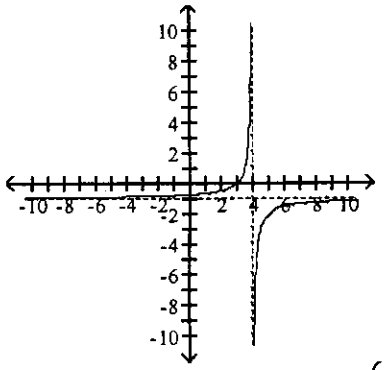
D) $-\sqrt{4}$, multiplicity 2; -2; $f(x) = (x + \sqrt{4})^2(x + 2)$

Graph the function, showing all asymptotes (those that do not correspond to an axis) as dashed lines. List the x- and y-intercepts.

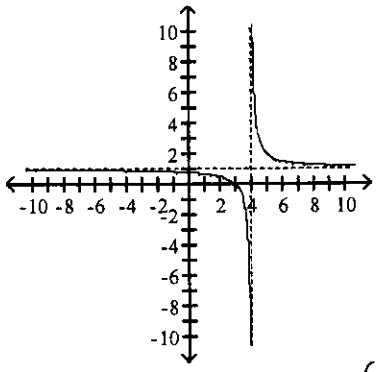
31) $f(x) = \dfrac{x-3}{x+4}$ 31) _____



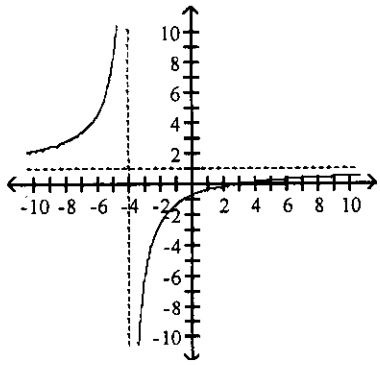
A) x-intercept: $(3, 0)$; y-intercept: $\left(0, -\dfrac{3}{4}\right)$;



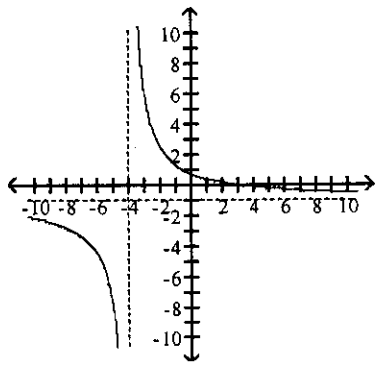
B) x-intercept: $(3, 0)$; y-intercept: $\left(0, \dfrac{3}{4}\right)$;



C) x-intercept: $(3, 0)$; y-intercept: $\left(0, -\dfrac{3}{4}\right)$;



D) x-intercept: $(3, 0)$; y-intercept: $\left(0, \dfrac{3}{4}\right)$;



Find the exact value by using a half-angle identity.

32) cos 22.5° 32) _____

A) $-\dfrac{1}{2}\sqrt{2-\sqrt{2}}$ B) $-\dfrac{1}{2}\sqrt{2+\sqrt{2}}$ C) $\dfrac{1}{2}\sqrt{2-\sqrt{2}}$ D) $\dfrac{1}{2}\sqrt{2+\sqrt{2}}$

Simplify the expression.

33) $\left(\frac{\cos x}{\sin x}\right)^2 - \frac{1}{\sin^2 x}$ 33) _____

A) 1 B) -1 C) $-\frac{1}{\sin^2 x}$ D) $\frac{1}{\sin^2 x}$

Solve the logarithmic equation.

34) $\ln x - \ln (x - 3) = \ln 2$ 34) _____

A) 6 B) -1 C) $\frac{3 \ln 2}{\ln 2 - 1}$ D) No solution

Solve the exponential equation.

35) $3x^2 + 5x = \frac{1}{81}$ 35) _____

A) 1 , -4 B) 1 , 4 C) -1 , 4 D) -1 , -4

Determine the equation of the line described. Put answer in the slope-intercept form, if possible.

36) Through (-6, -8), perpendicular to $-5x + 3y = 6$ 36) _____

A) $y = -\frac{5}{3}x - 58$ B) $y = \frac{3}{5}x + \frac{58}{5}$ C) $y = -\frac{3}{5}x - \frac{58}{5}$ D) $y = -\frac{3}{5}x$

Find the range of the given function.

37) $f(x) = 3x^2 + 24x + 45$ 37) _____

A) $[-3, \infty)$ B) $[-4, \infty)$ C) $(-\infty, 3]$ D) $(-\infty, 4]$

The terminal side of angle θ in standard position lies on the given line in the given quadrant. Find sin θ, cos θ, and tan θ.

38) $y = -6x$; quadrant II 38) _____

A) $\sin\theta = \frac{6\sqrt{37}}{37}$; $\cos\theta = -\frac{\sqrt{37}}{37}$; $\tan\theta = -6$

B) $\sin\theta = -\frac{\sqrt{37}}{37}$; $\cos\theta = \frac{6\sqrt{37}}{37}$; $\tan\theta = -6$

C) $\sin\theta = \frac{\sqrt{37}}{37}$; $\cos\theta = -\frac{6\sqrt{37}}{37}$; $\tan\theta = -6$

D) $\sin\theta = -\frac{6\sqrt{37}}{37}$; $\cos\theta = \frac{\sqrt{37}}{37}$; $\tan\theta = -6$

Solve the equation for the interval $[0, 2\pi)$.

39) $2\sin^2 x + \sin x = 1$ 39) _____

A) $\frac{\pi}{6}, \frac{3\pi}{2}$ B) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ C) $\frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ D) $\frac{\pi}{6}, \frac{5\pi}{6}$

Find an equation of an ellipse satisfying the given conditions.

40) Vertices: (0, -10) and (0, 10); length of minor axis: 6 40) _____

A) $\frac{x^2}{100} + \frac{y^2}{18} = 1$ B) $\frac{x^2}{9} + \frac{y^2}{100} = 1$ C) $\frac{x^2}{100} + \frac{y^2}{9} = 1$ D) $\frac{x^2}{100} + \frac{y^2}{9} = 10$

Answers

1 B
2 D
3 D
4 B
5 C
6 C
7 C
8 C
9 D
10 D
11 A
12 D
13 B
14 A
15 D
16 B
17 C
18 C
19 A
20 C
21 A
22 C
23 D
24 A
25 B
26 B
27 A
28 A
29 B
30 B
31 C
32 D
33 B
34 A
35 D
36 C
37 A
38 A
39 B
40 B