PROBLEMS ON NUMBER THEORY

I.37 For how many pairs (x,y) of integers is $y^2 - 2xy = 12$?
(a) 0 (b) 1 (c) 2 (c) 3 (d) 5

II.33 For which of the listed integers is it not possible to find integers x and y with $x^2 - y^2 = n$?
(a) 22 (b) 21 (c) -21 (d) 7 (e) 103

III.22 The number of solution pairs of positive integers of the equation $5x + 7y = 465$ is
(a) none (b) 9 (c) 13 (d) 43 (e) 66

III.35 Let n be the number of integer values of x such that $P = x^4 + 6x^3 + 11x^2 + 3x + 31$ is the square of an integer.
Then n = (a) 4 (b) 3 (c) 2 (d) 1 (e) 0

IV.29 The infinite sequence 11,111,1111,11111,... includes how many perfect squares? (a) 1 (b) 0 (c) 2 (d) more then two, but a finite number (e) infinitely many

V.34 If M and N are both perfect squares and $M - N = 29$ then $M + N =$
(a) 117 (b) 171 (c) 366 (d) 421 (e) 1048

VI.16 If the quadratic equation $x^2 + mx + n = 0$ has integer roots, and m and n are integers, then which of the following is not possible? (a) $mn > 0$ (b) n is odd and m is even (c) n is even and m is odd (d) n is odd and m is odd (e) n is even and m is even

VI.27 Given the equation $19m + 20n = 1987$, where m and n are positive integers, one solution is $m = 100$, $n = 1$. There is exactly one other solution for m and n, and in this case $m + n =$ (a) 25 (b) 30 (c) 33 (d) 37 (e) 39

X.23 If $x^2 + y^2 = z^2$ where x,y,z are positive integers and in arithmetic progression then (a) $z = 8x/3$ (b) $x + z$ is even (c) $y - x$ is odd (d) $x + y + z = 12$ (e) x is a prime number