PROBLEMS ON INTEGER DIVISORS

I.34 If n is an integer, which of the following could not divide both $n - 11$ and $n + 49$?
(a) 4 (b) 20 (c) 15 (d) 7 (e) 6

II.16 Consider the following three statements:
A: The product of three consecutive integers is always divisible by 3.
B: The product of four consecutive integers is always divisible by 4.
C: The product of five consecutive integers is always divisible by 5.
Which of these are true?
(a) all (b) none (c) A and B but not C (d) A and C but not B (e) only A

II.25 Denote by [n] the number 11....1 (n ones expressing a number in decimal notation. Thus for example [2] = 11 and [5] = 11111. Which one of the following is divisible by 7?
(a) [100] (b) [82] (c) [103] (d) [14] (e) [78]

III.3 How many positive integers, less than 100, are divisible by both 4 and 6?
(a) 4 (b) 6 (c) 8 (d) 10 (e) 12

IV.12 How many positive integers less than 100 have at least three different prime divisors?
(a) 4 (b) 6 (c) 8 (d) 5 (e) 10

IV.32 For positive integers M,N, where $M > N$, let G be the greatest common divisor of M and N. Then among the number pairs (I) N and $M + 2N$ (II) MN and $M + N$
(III) N and $M - N$ (IV) $M + N$ and $M - N$
the number G is also the greatest common divisor of
(a) I and III only (b) I,II, and IV only (c) III and IV only (d) III only (e) none of the pairs.

V.19 How many positive integers divide both of the integers 360 and 600? (a) 5 (b) 11 (c) 16 (d) 21 (e) 24

V.26 Give the smallest number among (a)-(e) which makes true the statement: In order to determine if the number 211 is a prime number it is sufficient to show 211 is not divisible by each prime up to and including
(a) 11 (b) 13 (c) 29 (d) 107 (e) 209

V.32 The smallest value of x for which $1,400x = N^3$ for some integer N is (a) 42 (b) 85 (c) 212 (d) 245 (e) 536 ?

VII.17 How many integers between 1 and 10,000 inclusive have each of 2,3,5,7 as a single divisor?
(a) 12 (b) 19 (c) 26 (d) 36 (e) 47

VIII.7 If n is a positive integer and $p = n(n + 1)(n + 2)$ then the largest integer which must divide p is
(a) 1 (b) 2 (c) 3 (d) 6 (e) 8 .

VIII.27 If 3 divides $(n + 2)$ then 3 must also divide which of the following: (a) $n^2 + 6n - 7$ (b) $n^2 + 3n$
(c) $n^2 + 4n$ (d) $n^2 - 4n - 5$ (e) $n^2 + 2n - 8$

IX.4 Let k be a positive integer and S the sum of k successive positive integers. Then S is divisible by k
(a) always (b) never (c) if and only if k is even
(d) if and only if k is odd (e) none of (a)-(d)

IX.7 If G is the greatest common divisor of 8547 and 4810 then the sum of the digits of g is
(a) 1 (b) 5 (c) 7 (d) 10 (e) 13

IX.11 How many integers between 1 and 100 can be written as a product of two different prime numbers?
(a) 15 (b) 19 (c) 22 (d) 26 (e) 30

X.21 There are how many pairs of different integers between 1 and 10 inclusive such that 3 divides the least common multiple of the pair? (a) 8 (b) 17 (c) 20 (d) 24 (e) 36