

PROBLEMS ON DIFFERENCE EQUATIONS

- I.13 Suppose $f(0) = 2$, $f(1) = 3$, $f(2) = 4$ and, for $u > 2$,
 $f(u) = f(u - 1) - 2f(u - 2) + f(u - 3)$. What is $f(5)$?
 (a) -1 (b) 9 (c) 0 (d) 7 (e) impossible to determine
 from given information
- I.35 Suppose $a_1 = 1$, $a_2 = 3$, and for $n \geq 2$, $a_n = 3a_{n-1} - 2a_{n-2}$.
 What happens to $(a_n)^{1/n}$ when n gets large?
 (a) gets close to 0 (b) gets close to 1 (c) gets close
 to 2 (d) gets close to 3 (e) gets arbitrarily large
- II.14 Suppose a function f is defined by $f(1) = 2$, $f(2) = 3$,
 $f(3) = 5$ and $f(n) = f(n - 1) + f(n - 2) + f(n - 3)$ for
 $n > 3$. What is $f(8)$?
 (a) 112 (b) 55 (c) 15 (d) > 500 (e) not enough
 information to tell
- II.23 Suppose a function f is defined by $f(1) = 1$, $f(2) = 2$,
 and $f(n) = 2f(n - 1) + f(n - 2)$ for $n > 2$. What is
 $f(100) - 2f(99) - 2f(97) - f(96)$?
 (a) -1 (b) 0 (c) 1 (d) ≤ -100 (e) ≥ 100
- III.20 If $f(x) = 3^{2x + 7}$ then $f(x + 1) - f(x) =$
 (a) 3^9 (b) $8f(x)$ (c) $f(2x) + 7$ (d) $9f(x)$ (e) 3
- III.31 Let f_1 be the fractional linear transformation
 $f_1(x) = (2x - 1)/(x + 1)$. Define $f_{n+1}(x) = f_1(f_n(x))$ for
 $n = 1, 2, 3, \dots$. Given that $f_{35} = f_5$, what is $f_{28}(x)$?
 (a) x (b) $1/x$ (c) $(x - 1)/x$ (d) $1/(1 - x)$ (e) none of
 these
- IV.16 Let $f(n)$ be a sequence such that $f(1) = 1$, $f(4) = 13$,
 and $f(n) = f(n - 1) + f(n - 2)$ for $n > 2$. Then $f(6) =$
 (a) 18 (b) 26 (c) 33 (d) many possible answers
 (e) no possible answers
- IV.25 If the integer N is initially assigned the value 1,
 and is then three successive times replaced by the square
 of one more than its value, then the resulting number is
 (a) 18 (b) 36 (c) 64 (d) 128 (e) 676
- V.27 Given the recurrence relation $f(0) = 2$, $f(1) = 1$,
 $f(n) = f(n - 1) - f(n - 2)$ for $n > 1$, then $f(100) =$
 (a) -101 (b) -2 (c) 2 (d) 1 (e) -1
- V.30 A student writes the number 1 on a chalkboard. Then in
 succession 10 students erase the number on the board and
 replace it by 1 less than three times the number on the
 board. The resulting number is
 (a) $3^{10} - 10$ (b) 2^{10} (c) $(10^3 - 1)/2$ (d) $(3^{10} + 1)/2$
 (e) $3(10^2 - 1)$
- V.31 On the set of positive real numbers let the transform-
 ation T be defined by $T(x) = 2/x$. Also let $T^{n+1}(x) =$
 $T(T^n(x))$, $n = 1, 2, 3, \dots$ where $T^1 = T$. Then $T^{10}(x) =$
 (a) $(2/x)^{10}$ (b) $2/x^{10}$ (c) $2^{10}/x$ (d) $(x/2)^{10}$ (e) x
- VI.21 Given $T(n) = 2n - 1$, $n = 1, 2, 3, \dots$, there are how many
 pairs of positive integers (k, l) such that
 $T^k(l) = 1025$ (where $T^k(n) = T(T^{k-1}(n))$)?
 (a) none (b) 1 (c) 3 (d) 5 (e) 10

- VI.23 Given the difference equation $f(n) = f(n-1) - f(n-2)$, if $f(1) = f(2) = 1$ then $f(1000) =$
 (a) -999 (b) -1 (c) 0 (d) 1 (e) 500 .
- VII.21 For each integer $n = 1, 2, \dots, 10$ if n is odd then person $(n+1)$ walks twice as far as person n , and if n is even then person $(n+1)$ walks $1/3$ as far as person n . If person 10 walks 1 mile then person 1 walks how many miles?
 (a) 3232323232 (b) $(1/2 + 3)^5$ (c) $(3/2)^{10}$ (d) $(2/3)^{10}$
 (e) $81/32$
- VIII.16 If $f(n) = 1/n - 1/(n+2)$ then $f(1) + f(2) + \dots + f(9) =$
 (a) $47/33$ (b) $69/51$ (c) $87/26$ (d) $72/55$ (e) $17/12$
- VIII.22 If $f(0) = 1$ and $f(n) = 2f(n-1)$ for n odd, $f(n) = \log_4 f(n-1)$ for n even then $f(10)$ is
 (a) 0 (b) 1 (c) $\log_4 512$ (d) 2 (e) undefined
- VIII.25 If $f(1) = 1$, $f(n) = f(n/2) + 1$ if n is even and $f(n) = 1/f(n-1)$ if n is odd then $f(50) =$
 (a) $9/7$ (b) $99/75$ (c) $39/15$ (d) $427/5$ (e) $1050/27$
- IX.24 Let $f(n)$ be a sequence of numbers defined by $f(1) = 1$, $f(n) = f(n-1) + 2$ if n is even, and $f(n) = f(n-1)/2$ if n is odd. Then $4 - f(100) =$
 (a) $(1/2)^{100}$ (b) 2^{98} (c) $(1/2)^{98}$ (d) $(1/2)^{49}$
 (e) $(1/4)^{29}$
- X.27 For $f(x)$ a real valued function define $f^2(x) = f(f(x))$ and $f^{n+1}(x) = f(f^n(x))$ for $n = 2, 3, \dots$. Then if $f(x) = x + 3$, $f^{10}(x) =$ (a) $10x + 30$ (b) $x + 30$ (c) $10x + 3$
 (d) $x^{10} + 3^{10}$ (e) $x^{10} + 30$
- X.13 If $f(0) = 0, f(1) = 1$, and $f(n) = (f(n-1) - f(n-2))/2$ for $n > 1$ then $f(101) - 2/3 =$ (a) $5/202$ (b) $3/101^2$
 (c) $1/3(2)^{100}$ (d) $1/2^{101}$ (e) $-1/100^2$