

PROBLEMS ON COUNTING -

- I.38 If N is the number of 5 card poker hands which contain exactly one pair (no other pairs or triples) then the highest power of 2 that divides N is
 (a) 3 (b) 5 (c) 7 (d) 8 (e) 9
- II.22 In a class of 40 students every student takes at least one of the subjects Math, Science, and English. Of these subjects 27 take Math, 15 take Math and Science, 19 take Math and English, and 17 either take only Math or else take all three. How many students take only Math?
 (a) 5 (b) 6 (c) 7 (d) 8 (e) 9
- III.13 A "full house" poker hand consists of a triple and a pair e.g. three jacks and two eights. How many different full house hands are possible in a standard deck of 52 cards? (a) 64 (b) 3744 (c) 5280 (d) 6536 (e) 27,264
- IV.23 For N a positive integer let $T(N) = N/2$ if N is even and $T(N) = (N - 1)/2$ if N is odd. For how many positive integers is $T(T(T(N))) = 17$?
 (a) 1 (b) 2 (c) 4 (d) 8 (e) 16
- IV.26 How many positive integers less than 1,000 have the sum of its digits equal to 10?
 (a) 47 (b) 55 (c) 63 (d) 72 (e) 81
- V.1 Given a set of 5 points in the plane, no two of which are collinear, how many different triangles can be formed using the 5 points? (a) 5 (b) 8 (c) 10 (d) 15 (e) 125
- V.18 A set of 8 elements has how many different subsets with 6 or more elements?
 (a) 37 (b) 52 (c) 60 (d) 84 (e) 224
- V.23 Let T be the triangle in the Cartesian plane with vertices $(0,0)$, $(1,0)$ and $(1,3)$. How many points (x,y) are there inside T (do not include points on the sides of T) where both x and y may be written in the form p/q , $q < 5$, where p and q are positive integers.
 (a) 26 (b) 40 (c) 40 (d) 62 (e) 81
- V.33 How many integers between 100 and 999 have their digits in increasing order (i.e. if the integer is 'xyz' then $x < y < z$). (a) 78 (b) 84 (c) 106 (d) 121 (e) 186
- VI.13 Given the triangle in the Cartesian plane with vertices $(0,0)$, $(0,10)$ and $(20,0)$, how many points (m,n) are interior to the triangle if m and n are both integers. (do not include points on the perimeter of the triangle)
 (a) 72 (b) 81 (c) 92 (d) 100 (e) 108
- VI.24 How many 8 letter words, consisting only of letters a and b , are there which do not have two consecutive a 's? (i.e. the word $abbbabab$ is counted but the word $baababab$ is not counted) (a) 13 (b) 21 (c) 36 (d) 42 (e) 55
- VI.29 Let $S = \{1,2,3,\dots,21\}$ be the set of all integers from 1 to 21 inclusive. In how many ways can two different integers be selected from S so that the sum is even?
 (a) 92 (b) 100 (c) 108 (d) 116 (e) 134

- VII.7 John wishes to simulate the drawing of a card from a standard deck of 52 cards by repeatedly tossing a coin. What is the fewest number of times the coin must be tossed in order that different cards may be represented by different sequences of heads and tails?
(a) 6 (b) 9 (c) 12 (d) 18 (e) 26
- VII.13 Five circles are drawn in the plane, dividing the plane into regions (two points are in the same region if they are the end points of a line segment which does not intersect any of the circles). What is the largest possible number of regions?
(a) 32 (b) 28 (c) 22 (d) 15 (e) 8
- VII.19 Given the equation $4x + 3y = 529$ the number of solution pairs (x,y) where x and y are positive integers is (a) 7 (b) 17 (c) 23 (d) 33 (e) 43
- VII.22 A round robin tournament is one in which each team plays each other team exactly once. How many games are played in a round robin tournament of 8 baseball teams?
(a) 64 (b) 56 (c) 32 (d) 28 (e) 15
- VII.26 A robot can walk in steps of 1,2, or 3 feet. In how many ways can a robot walk 6 feet?
(a) 11 (b) 16 (c) 24 (d) 30 (e) 36
- IX.6 Eight baseball teams play in a double elimination tournament (a team is eliminated when it has lost two games). The maximum number of games in the tournament is
(a) 15 (b) 18 (c) 21 (d) 24 (e) 26
- X.6 How many integers between 1 and 1000 have the sum of the digits equal to 5? (a) 48 (b) 39 (c) 33 (d) 25 (e) 21