

PROBLEMS ON THE CONGRUENCE RELATION

- III.17 Given three piles of coconuts, if $\frac{1}{5}$ of the total number of coconuts is in the first pile, several sevenths are in the second pile, and 12 coconuts are in the third pile, what is the total number of coconuts?
 (a) 105 (b) 120 (c) 140 (d) 420 (e) 700
- IV.27 For positive integers A and B let $A \bmod B$ denote the remainder when A is divided by B (e.g. $23 \bmod 4 = 3$). If $N = abc$ is a 3 digit number, where a,b,c are digits, then $c =$
 (a) $(N \bmod 100) \bmod 10$
 (b) $N \bmod 1000 - N \bmod 100 + N \bmod 10$
 (c) $N \bmod 100 + N \bmod 10$
 (d) $(N/N \bmod 100)/N \bmod 10$
 (e) $(N \bmod 100) \times (N \bmod 10)$
- V.28 Given a pile of x coconuts, if the pile is subdivided into 3 equal piles then there is 1 coconut left over. If it is subdivided into 5 equal piles then there are 2 coconuts left over, and if it is divided into 7 equal piles then there are 3 coconuts left over. If $x < 100$ then the sum of the digits of the integer x is
 (a) 3 (b) 4 (c) 6 (d) 7 (e) 9
- VI.10 If for positive integers M and N, $M \bmod N$ is the remainder from the division of M by N then $46 \bmod ((60 \bmod 31) \bmod 11)$ equals
 (a) 13 (b) 9 (c) 8 (d) 4 (e) 0.
- VII.28 If M,N are integers then $M = N \bmod D$ provided $M - N$ is divisible by D. Given that $M = N \bmod D$ and $P = Q \bmod D$ then of (I) $M + P = (N + Q) \bmod D$ (II) $MP = NQ \bmod D$ (III) $MQ = NP \bmod D$ (IV) $MN = PQ \bmod D$ which are not necessarily true? (a) II,III (b) I,IV (c) III only
 (d) IV only (e) all must be true
- VIII.29 Let M,N,P be positive integers, and $M \bmod N$ be the remainder of the division of M by N. If $M \bmod N = 3$ and $N \bmod P = 5$ then $M \bmod P$ could be which of the following numbers? (a) 15 (b) 19 (c) 27 (d) 2 (e) 13
- X.8 For positive integers by a mod c is meant the division of a by b gives a remainder c. Given $x \bmod 7 = 2$, $y \bmod 7 = 5$, and $z \bmod 7 = 3$ then $xyz \bmod 7 =$ (a) 0 (b) 1 (c) 2
 (d) 3 (e) 4