Theta Individual Test
2000 Mu Alpha Theta National Convention

> The abbreviation NOTA denotes "None of These Answers"

1. It is given that $f(x) = x^2 - 3x + 12$ and $g(x) = 2x + 6$, and $g^{-1}(x)$ is the inverse relation to g. Give the value of $g^{-1}(f(2))$.

   A. 2 B. 4
   C. 10 D. 26 E. NOTA

2. Simone intended to multiply a number A by 9 but instead divided it by 9. She then meant to subtract 10 but instead added 10. After these mistakes the result was 320. Which is true of the original number A?

   A. $1,000 < A < 2,000$
   B. $2,000 < A < 10,000$
   C. $10,000 < A < 20,000$
   D. $20,000 < A < 30,000$
   E. NOTA

3. A large rectangle is partitioned into four rectangles of nonoverlapping areas, by two segments parallel to its sides, as shown (not drawn to scale). The areas in square units of the resultant rectangles are shown. What is the area of the unknown rectangle, in square units?

12	21
?	35

   A. 15
   B. 20
   C. 28
   D. 30
   E. NOTA

4. If $3^x = m$ and $2^x = n$ then which is equivalent to 6^x? $(x > 0)$

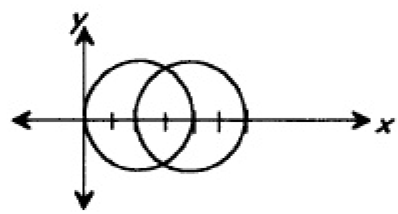
   A. $\log(m+n)$ B. mn
   C. $\log m + \log n$ D. $m+n$
   E. NOTA

5. Which expression is equivalent to $4^2 \cdot 9^2 \cdot 4^8 \cdot 9^8$?

   A. 36^{16} B. 13^{10}
   C. 6^{20} D. 6^{40} E. NOTA

6. The lines $y = 3x + 2$ and $y = 2$ and $x = 2$ bound a triangular region. What is the area of that region, in square units?

   A. 4 B. 6
   C. 8 D. 16 E. NOTA

7. 

   The two congruent circles shown have diameters on the x-axis. One has center at (2,0) and the other has center at (4,0). The first circle is tangent to the y-axis. Which is a y-coordinate of an intersection point of the two circles?

   A. 3 B. $\sqrt{5} - 1$
   C. $\sqrt{3}$ D. $\sqrt{2}$ E. NOTA

8. A test has ten questions. A correct answer is awarded 3 points, and an incorrect answer is awarded -1 point. Answers left blank are awarded zero points. Which of the following scores is not possible for the test? Each answer is in points, not percents.

A. 23 B. 25
C. 26 D. 27 E. NOTA

9. If $a\Psi b = \frac{1}{\log b} + \frac{1}{\log a}$ then which is equivalent to $a\Psi b$ for all $a > 1,\ b > 1$?

A. $\frac{\log(a+b)}{(\log a)(\log b)}$ B. 1
C. $\frac{\log(ab)}{\log(a+b)}$ D. $\frac{\log(ab)}{(\log a)(\log b)}$
E. NOTA

10. If A and B are digits (base ten) then if

$$\begin{array}{r} A\ 7\ 3 \\ -\ \underline{4\ 8\ B} \\ B\ 8\ A \end{array}$$

find the value of A+B.

A. 13 B. 14 C. 11 D. 12 E. NOTA

11. A piece of graph paper is folded once so the point (0, 1) is matched with the point (3, 0). Which point will lie on the fold?

A. (1, 0) B. (0, −4)
C. (2, 1) D. $(\frac{4}{3}, -4)$ E. NOTA

12. The range of the relation f is integers. For $x \geq 1$ $f(x) = f(x-1) \bullet f(x-2)$ and $f(1) = 14$. If $f(-1)$ and $f(0)$ exist, which could not be $f(3)$?

A. 49 B. 42
C. 14 D. 70 E. NOTA

13. In a coordinate plane, the line y=5 is rotatec 30 degrees clockwise about the point (0, 5). give the x-intercept of the rotated line.

A. $(5,0)$ B. $(5\sqrt{3},0)$
C. $(-5,0)$ D. $(-5\sqrt{3},0)$ E. NOTA

14. A rectangular piece of cardboard is used to create a cylinder's lateral surface. If the rectangle is 8 cm by 10 cm and the cylinder is 8 cm high, then find the area in square cm of the base that is made for this cylinder.

A. $\frac{5}{\pi}$ B. $\frac{10}{\pi}$
C. $\frac{25}{\pi}$ D. $\frac{15}{\pi}$ E. NOTA

15. The average of eight integers is -5. What is the maximum number of the eight integers that could be greater than 20?

A. 2 B. 3
C. 6 D. 7 E. NOTA

16. Juanita wishes to construct out of a 80 inch by 80 inch piece of construction paper a pyramid with a regular hexagonal base and congruent isosceles triangles on each lateral face. She will cut one polygon and fold it to make the pyramid. If the base of the pyramid will have 10 inch edges, which could not be the length of one side of a lateral face?

A. $10\sqrt{2}$ inches
B. 9 inches
C. $\sqrt{102}$ inches
D. $10+\sqrt{2}$ inches
E. NOTA

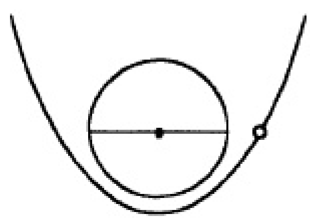
17. $3x^{-1} \bullet -3^{2} \bullet 3^{\frac{3}{2}} = a$ for $x = 9$. Which is equivalent to a ?

A. $-9\sqrt{3}$ B. $6\sqrt{3}$
C. $-\sqrt{3}$ D. $\sqrt{3}$ E. NOTA

18. A regular hexagon is in plane H. How many distinct circles in plane H can be drawn so that the endpoints of a diameter are on vertices of the hexagon?

A. 13 B. 15
C. 16 D. 17 E. NOTA

19. A comet approaces a planet that is perfectly spherical. The comet follows a parabolic path with the planet's center at the focus of the parabola. The radius of the planet is 5000 miles. As the comet passes the vertex of the path, it is 100 miles above the planet. As the comet passes through the line containing the diameter that is parallel to the parabola's directrix, how far is the comet above the planet's surface?



A. 10400 miles B. 5200 miles
C. 1290 miles D. 290 miles
E. NOTA

20. Right triangle ABC has sides which are consecutive even integers. $\overline{AB}$ is the shortest side. If $\overline{BD}$ is the altitude to the hypotenuse (D is on $\overline{AC}$) then find the length AD.

A. 3 B. 3.6
C. 4 D. 6.4 E. NOTA

21. The graphs of $f(x) = x^3 + ax^2 + bx + c$ and $g(x) = x^3 + bx^2 + ax + c$ for $(abc \neq 0, a \neq b)$ intersect in two distinct points. Give the sum of the x-coordinates of these points.

A. 1 B. 0
C. -1 D. $a+b$ E. NOTA

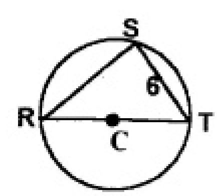
22. Jerome and his sister were both born on January 1 in different years. Let f(x) denote the age that Jerome is at the year x, for $x \geq$ his birth year plus two years. Let g(x) denote the age his sister was when Jerome was $f(x)-2$ years old. If $g(1990) = 10$ and $f(x) = g(x) + 6$ then how much older is Jerome than his sister?

A. 2 years B. 4 years
C. 6 years D. 8 years
E. NOTA

23. The graphs of $y = x^3 + ax^2 + bx + c$ $(abc \neq 0)$ and $y = -x^3 + dx^2 + ex + f$ $(def \neq 0)$ intersect in k distinct points. Let L be the least number possible for k and G be the greatest possible value for k. What is the value of the sum L+G ?

A. 2 B. 3
C. 4 D. 5
E. NOTA

24. The radius of circle C is 5 and a triangle RST is inscribed as shown, with one side a diameter of C and ST=6. What is the area of ΔRST ?



A. 30
B. 24
C. 15
D. 12
E. NOTA

25. If $f(x) = x^{0.5(x+1)}$ for $x > 0$ then give the value of $(f(4))^{\frac{1}{2}}$.

A. 32 B. 128
C. $4\sqrt{2}$ D. $2\sqrt{2}$ E. NOTA

26. Each of the letters below represent a different digit from 1 through 9, inclusive.

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  A B C        A > B > C
− D E 3
  B 6 4
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Which of the digits from 1 through 9 inclusive, are <u>not</u> represented in the problem?

A. 2 B. 1
C. 9 D. 5 E. NOTA

27. Find the value of k so that the remainder when $x^3 - 4x^2 + k$ is divided by $(x-1)$ is 2.

A. 3 B. 5
C. 7 D. 9 E. NOTA

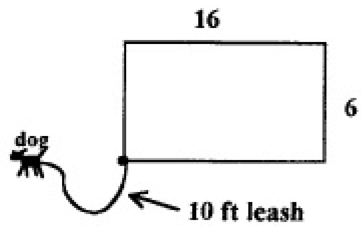
28. A population of bunnies triples every year, when measured on the first day of the year. Originally there were 4 bunnies and in 1990 on January 1,there were $4(3^{90})$ bunnies. How many bunnies were there in 1989, January 1?

A. $2(3^{45})$ B. $2(3^{89})$
C. $4(3^{45})$ D. $4(3^{89})$
E. NOTA

29. A woman has three children; at least two are girls. What is the probability that she has two girls and a boy?

A. $\frac{1}{3}$ B. $\frac{1}{2}$
C. $\frac{3}{4}$ D. $\frac{2}{3}$ E. NOTA

30. A dog is leashed to the corner of a rectangular building. The leash is attached s that when he roams, it is horizontal when ta (parallel to the ground). If the building is 16 feet by 6 feet and the leash is 10 feet long. What is the surface area on the ground that the dog can roam? Assume that the dog does not dig or become airborne. Ignore the length of the dog.



A. 75π sq. ft. B. 79π sq. ft.
C. 68π sq. ft. D. 128π sq. ft.
E. NOTA