Equations and Inequalities-Theta
2000 National Mu Alpha Theta Convention

1) Find r if $\frac{r}{6}+\frac{r}{3}+\frac{r}{2}=1$.

A) 1 B) ½ C) 2 D) 12/11 E) NOTA

2) Find the sum of the reciprocals of the solutions of $6x^2-5x+1=0$.

A) -1 B) 5/6 C) 5 D) -1/5 E) NOTA

3) Solve for r: $\frac{6r+2}{3-r}=4$.

A) -1 B) 1 C) 3 D) 10/7 E) NOTA

4) Solve for x: $\frac{3}{3+\frac{3}{3+x}}=3$.

A) -1 B) 1 C) 9/2 D) -9/2 E) NOTA

5) Find the smallest integer n such that $\frac{1}{2n+3}<-\frac{1}{7}$.

A) -4 B) -5 C) -6 D) -7 E) NOTA

6) Let p/q be a ratio of positive integers such that no p and q have no common divisors besides 1 and $\frac{p}{q}=\sqrt{3}$. Squaring both sides gives us $\frac{p^2}{q^2}=3$, so that $p^2=3q^2$. Therefore, p is divisible by 3. Hence we can write $p=3p_1$, so our equation is now $9p_1^2=3q^2$, or $3p_1^2=q^2$. Hence, q is also divisible by 3, which contradicts our assumption that p and q have no common divisors besides one. This argument proves

A) that $\sqrt{3}$ is rational B) that $\sqrt{3}$ is irrational C) that 3 is prime
D) Nothing E) NOTA

7) What real values of z form a complete solution set to $3|z-2|+3\geq 2-|z|$?

A) none B) $0\leq z\leq 2$ C) $z\geq 2$ D) all real values of z E) NOTA

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8) Solve for real x: $\frac{3}{x^2} \le 27$.

A) $-1/3 \le x \le 1/3$ B) $x \le -1/3$ or $x \ge 1/3$
C) $-3 \le x \le 3$ D) $x \le -3$ or $x \ge 3$
E) NOTA

9) Find k such that $k + 2k + 3k + 4k + \ldots + 20k = 2730$.

A) 11 B) 12 C) 13 D) 14 E) NOTA

10) For how many real values of w does $3^w - 2^w = 53$?

A) 0 B) 1 C) 2 D) infinitely many E) NOTA

11) Find all y such that $3 - 2|y - 2| \ge 2 - y$.

A) $y \le 1, 2 \le y \le 5$ B) $2 \le y \le 5$ C) $1 \le y \le 5$ D) $y \le 1$ E) NOTA

12) Find the sum of all distinct z such that $\frac{3}{z} + \frac{z}{3} = -2$.

A) -6 B) -3 C) 0 D) 3 E) NOTA

13) How many different integers m satisfy $\frac{5+m}{5-2m} \ge 1$?

A) 3 B) 4 C) 5 D) 6 E) NOTA

14) Find the smallest positive even integer a such that for some integers b and c we can write $a^2 = 2c^2 - b^2$?

A) No such a exists B) 2 C) 4 D) 12 E) NOTA

15) Find all values of y which satisfy $15y^2 - 14y - 49 = 0$.

A) 5/3, -1/21 B) 7/3, -7/5 C) 5/3, -1 D) 5/3, 7/5 E) NOTA

16) Find the sum of the third powers of the four fourth roots of 4.

A) $8\sqrt{2}$ B) $-8\sqrt{2}$ C) 0 D) 4 E) NOTA

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17) Find $x + y$ if

$$3x + \frac{2}{y} = 9$$

$$-2x + \frac{7}{y} = 19$$

A) 4 B) 4/3 C) 2/3 D) 2 E) NOTA

18) How many pairs of positive integers (m,n) satisfy $4m + 3n = 129$?

A) 7 B) 8 C) 9 D) 10 E) NOTA

19) What is the smallest possible value of $3x^4-2x^2+7$ for real values of x?

A) 7 B) 6 C) 20/3 D) 62/9 E) NOTA

20) If a and b are positive integers such that $\frac{1}{a}+\frac{1}{b}=\frac{1}{7}$, then what is the largest possible value of $\min(a,b)$, where $\min(a,b)$ is defined as the smaller of a and b?

A) 6 B) 7 C) 13 D) 14 E) NOTA

21) It takes 8 people 5 days to build 3 huts. How many days will it take to build one hut if one person works on the first day and one person joins at the beginning of each day thereafter?

A) 4 2/3 B) 5 C) 4 13/15 D) 4 4/5 E) NOTA

22) When factored into as many polynomials as possible of degree one or greater with real coefficients, how many factors does x^5+1 have?

A) 2 B) 3 C) 4 D) 5 E) NOTA

23) Solve for y: $\dfrac{4}{4+\frac{4}{y}} = \dfrac{3}{3-\frac{3}{y}}$.

A) 0 B) 1 C) $-\sqrt{2}$ D) no solution E) NOTA

24) Find the sum of all the solutions of $x^3 - 1 = 0$ which have nonzero imaginary parts.

A) 0 B) 1 C) -1 D) $\sqrt{3}$ E) NOTA

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25) Given that $a^2 + b = 6$, and a and b are both positive, what is the largest possible value of a^4b^2?

A) 81 B) 64 C) 100 D) 75 E) NOTA

26) Find the sum of the positive values of x which satisfy $6x^4 + 19x^3 - 259x^2 + 489x - 135 = 0$.

A) 10/3 B) 17/6 C) 35/6 D) 3 E) NOTA

27) We define the length of interval (a,b) to be $b - a$. Find the sum of the lengths of all intervals which together form a non-overlapping, complete solution set to $\frac{1}{y-2} - 2 > \frac{1}{y}$.

A) 2 B) 3/2 C) $2\sqrt{2} - 2$ D) $\sqrt{5}$ E) NOTA

28) Find $x + y + z$ if

$$x\sqrt{y} = 108$$
$$y\sqrt{z} = 80\sqrt{3}$$
$$z\sqrt{x} = 225\sqrt{3}$$

A) 118 B) 124 C) 126 D) 132 E) NOTA

29) Given that $a_1^2 + a_2^2 + a_3^2 = 7$, find the maximum possible value of $3a_1 + 2a_2 + a_3$.

A) $7 + 2\sqrt{2}$ B) $7\sqrt{2}$ C) $2\sqrt{21}$ D) $9\sqrt{3}$ E) NOTA

30) Find the positive difference between the largest and smallest real values of x which satisfy $x^4 + x^3 - 28x^2 + x + 1 = 0$.

A) $\sqrt{21} + \sqrt{2} + \frac{9}{2}$ B) $\frac{\sqrt{21}}{2} + 2\sqrt{2} - 2$ C) $\sqrt{21} + 2\sqrt{2} - 2$

D) $\frac{\sqrt{21}}{2} + 2\sqrt{2} + \frac{11}{2}$ E) NOTA