

2000 Mu Alpha Theta National Convention
GEOMETRY

Figures are not drawn to scale. NOTA means None Of The Above.

- A circle with a 6 inch radius is "folded over" so that the circle passes through its own center. Assuming that this were possible, and that it was 100% accurate, what is the length of the chord that is formed when the fold is creased?
 - 6 inches
 - $6\sqrt{2}$
 - $6\sqrt{3}$
 - cannot be determined from the given information
 - NOTA
- The bisector of an angle divides the angle into two angles, each of which is 30° less than the complement of the original angle. Find the measure of the original angle.
 - 30°
 - 40°
 - 50°
 - 60°
 - NOTA
- An angle is bisected and then one of those halves is bisected and then one of those halves is bisected. Now one of the angles that has been formed with the smallest measure is a supplement of the original angle. What is the measure of that original angle?
 - 40°
 - 140°
 - 144°
 - 160°
 - NOTA
- The acute angle in a right triangle formed by the hypotenuse 50 cm and a leg 14 cm is bisected. Find the length of that bisecting segment from the vertex of the acute angle to the opposite leg.
 - 22.5
 - 20.5
 - 17.5
 - 15.5
 - NOTA
- Consider a triangle with sides of lengths 13 cm, 14 cm, and 15 cm. Find the length of the altitude to the 14 cm side.
 - 12 cm
 - 12.5 cm
 - $15\sqrt{5}$ cm
 - $11\sqrt{5}$ cm
 - NOTA
- A circle with an 8 cm radius has a diameter extended on both ends. One side is extended 2 cm. Perpendicular tangent segments (from the same external point) are drawn that meet the diameter at its extended endpoints. How far was the diameter extended on the other side?
 - $4\frac{1}{2}$ cm
 - $5\frac{1}{3}$ cm
 - $6\frac{1}{2}$ cm
 - $6\frac{1}{3}$ cm
 - NOTA
- An isosceles trapezoid with bases 10 inches and 40 inches is circumscribed about a circle. Find the length of the radius of the circle.
 - 10 inches
 - 15 inches
 - 20 inches
 - cannot be determined from the given information
 - NOTA
- Find the length of the radius of the circle with the equation:
$$3x^2 + 3y^2 - 18x + 12y - 153 = 0$$
 - 6
 - 8
 - 10
 - 15
 - NOTA

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9. A cube has its total surface area equal to its volume. Find the length of a diagonal of the cube.
- A. $6\sqrt{2}$
B. $8\sqrt{2}$
C. $6\sqrt{3}$
D. $8\sqrt{3}$
E. NOTA
10. How many diagonals does a convex polygon have if the sum of the measures of its interior angles is 2700?
- A. 90
B. 119
C. 152
D. 180
E. NOTA
11. How many of the following are ways to prove a quadrilateral is a parallelogram?
- ◆ both pairs of opposite sides equal in length
 - ◆ one pair of opposite sides parallel and equal
 - ◆ both pairs of opposite sides parallel
 - ◆ both pairs of opposite angles equal
 - ◆ both diagonals congruent
 - ◆ diagonals perpendicular
 - ◆ both diagonals bisect each other
 - ◆ a diagonal forms two congruent triangles
 - ◆ both diagonals bisect the angles to which they are drawn
 - ◆ segment joining the midpoints of both pairs of opposite sides bisect each other
 - ◆ segments joining the midpoints of both pairs of opposite sides are perpendicular
- A. 5
B. 6
C. 8
D. 9
E. NOTA
12. Consider two circles in a plane and their positional relationships. Now from all these possible placements, consider the number of common tangents that can exist for each one. Add these possibilities and use that number as the length of the hypotenuse of a right triangle. What is the length of the median drawn to that hypotenuse in the right triangle you just made?
- A. 4
B. 4.5
C. 10
D. cannot be determined from the given information
E. NOTA
13. Two tangent segments to the same circle meet to form a 60° angle. A secant segment to that circle from that same exterior point is drawn. If the secant segment and the diameter of the circle are both 8 cm in length, find the length of the chord of the secant segment.
- A. 6 cm
B. 4 cm
C. 3 cm
D. 2 cm
E. NOTA
14. A sphere with a radius of 6 cm is inscribed in a can. (That is the sphere is tangent to the can at the top, the bottom, and on the side around a great circle of the sphere.) How much space in the interior of the can is not used by the sphere?
- A. $144\pi \text{ cm}^3$
B. $288\pi \text{ cm}^3$
C. $360\pi \text{ cm}^3$
D. $423\pi \text{ cm}^3$
E. NOTA

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15. The legs of a right triangle are 12 and 35. Find the length of the altitude to the hypotenuse.
- A. $\frac{375}{37}$
B. $\frac{400}{37}$
C. $\frac{420}{37}$
D. $\frac{425}{37}$
E. NOTA
16. Two tangent segments to the same circle meet to form a 40° angle. What is the measure of the minor arc determined by the points of tangency?
- A. 100°
B. 120°
C. 140°
D. 160°
E. NOTA
17. The endpoints of a circle's diameter are located at $(-4, -7)$ and $(10, -1)$. Which of the following is the equation of the circle?
- A. $x^2 + y^2 - 6x + 8y - 33 = 0$
B. $x^2 + y^2 + 6x - 8y - 58 = 0$
C. $x^2 + y^2 + 6x - 8y - 51 = 0$
D. $x^2 + y^2 - 3x + 4y - 58 = 0$
E. NOTA
18. Consider the following statements about similar polygons and determine which must be true.
- I. similar polygons can never be congruent
II. all regular polygons are similar
III. all corresponding lengths of similar polygons must have the same ratio
- A. I and II only
B. II and III only
C. I and III only
D. I, II, and III
E. NOTA
19. A chord that is 8 cm from the center of a circle with a radius of 17 cm intersects another chord of the circle dividing it into segments that are 9 cm and 24 cm. Find the length of the longer segment of the original chord.
- A. 12 cm
B. 15 cm
C. 18 cm
D. 21 cm
E. NOTA
20. If an angle of a triangle is 30° and the sides of the triangle that form the angle have a measure of 8 inches and 20 inches, what is the area of the triangle?
- A. 160 square inches
B. 80 square inches
C. 40 square inches
D. cannot be determined from the given information
E. NOTA
21. Two chords of a circle are perpendicular and divide each other into segments that are 6 cm and 8 cm for one chord and 4 cm and 12 cm for the other chord. What is the length of a radius of this circle?
- A. $\sqrt{113}$ cm
B. $\sqrt{65}$ cm
C. $8\sqrt{3}$ cm
D. $\frac{15}{3}\sqrt{3}$ cm
E. NOTA
22. Consider a rectangle with length 20 inches and width 16 inches. A corner is cut from the rectangle and the lengths of the sides of the pentagon that has been formed are 16 inches, 20 inches, 4 inches, $2\sqrt{85}$ inches, and 6 inches, sequentially. What is the area of the pentagon?
- A. 236 square inches
B. $80\sqrt{3}$ square inches
C. 152 square inches
D. $118\sqrt{3}$ square inches

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23. Through the vertex angle of isosceles $\triangle ABC$ with legs \overline{AB} and \overline{AC} equal to 15 cm and base \overline{BC} equal to 24 cm, a line parallel to the base is constructed. A point, P, is located on this line 20 cm from the vertex of the vertex angle. Find the area of the triangle PBC.
- A. 96 cm^2
 B. 108 cm^2
 C. 120 cm^2
 D. 144 cm^2
 E. NOTA
24. In quadrilateral SPOR the diagonal \overline{PR} is drawn. If $m\angle S = 100^\circ$, $m\angle SPR = 30^\circ$, $m\angle RPO = 95^\circ$, and $m\angle O = 60^\circ$. Which of the following is the longest side of the quadrilateral?
- A. \overline{PR}
 B. \overline{SP}
 C. \overline{OR}
 D. cannot be determined from the given information
 E. NOTA
25. Find the area of a regular hexagon with an apothem of 18 cm.
- A. $648\sqrt{3}$ square cm
 B. $486\sqrt{3}$ square cm
 C. $1296\sqrt{3}$ square cm
 D. $324\sqrt{3}$ square cm
 E. NOTA
26. In quadrilateral DROP, diagonal \overline{DO} divides diagonal \overline{RP} into two segments of 11 cm and 19 cm. Diagonal \overline{RP} divides diagonal \overline{DO} into segments that are 7 cm and 12 cm. If the diagonals are perpendicular, find the area of the quadrilateral DROP
- A. 305 square cm
 B. 292.5 square cm
 C. 285 square cm
 D. 293 square cm
 E. NOTA
27. In trapezoid TRAP with $\overline{TR} \parallel \overline{AP}$. $PT = TR = RA$. If $m\angle TRA = 140^\circ$, find $m\angle PRA$.
- A. 120°
 B. 110°
 C. 100°
 D. 90°
 E. NOTA
28. Quadrilateral MATH is inscribed in circle O. If $m\widehat{MA} = 120^\circ$ and $m\widehat{MH} = 80^\circ$, find $m\angle HMA$.
- A. 100°
 B. 90°
 C. 80°
 D. 70°
 E. NOTA
29. Find the length of the hypotenuse of a right triangle with an acute angle of 50° and the side opposite that acute angle 38 mm to the nearest mm.
- | measure | sin | cos | tan |
|------------|-------|-------|--------|
| 40° | .6428 | .7660 | 0.8391 |
| 45° | .7071 | .7071 | 1.0000 |
| 50° | .7660 | .6428 | 1.1918 |
- A. 45 mm
 B. 48 mm
 C. 50 mm
 D. 52 mm
 E. NOTA
30. Consider the following points of concurrency. Which of them must lie on the interior of a triangle?
- I. centroid
 II. circumcenter
 III. incenter
 IV. orthocenter
- A. I only
 B. I and II only
 C. I and III only
 D. I, III, and IV only
 E. NOTA