Equations and Inequalities-Alpha
2000 National Mu Alpha Theta Convention

1) Solve for x: $\frac{3x+2}{3-x} = 7$.

A) 4.75 B) 5.75 C) 2.3 D) 1.9 E) NOTA

2) Find the sum of the reciprocals of the solutions to the equation $8x^2 - 7x + 1 = 0$.

A) 7 B) -7 C) 7/8 D) -8/7 E) NOTA

3) Solve for r: $\frac{4}{r^4} \le 64$.

A) $-1/2 \le r \le 1/2$ B) $r \le -1/2$ or $r \ge 1/2$
C) $-2 \le r \le 2$ D) $r \le -2$ or $r \ge 2$
E) NOTA

4) Find the sum of all real x which satisfy $\frac{3}{3-x^{10}} = \frac{4}{4+x^{10}}$.

A) 4/3 B) 12 C) -1 D) 0 E) NOTA

5) Find all real y such that $\frac{|y|}{(|y|+1)(y-1)} = 1$.

A) $\frac{1-\sqrt{5}}{2}$ B) $\frac{1+\sqrt{5}}{2}$ C) $\frac{3+\sqrt{5}}{2}$ D) $\frac{3-\sqrt{5}}{2}$ E) NOTA

6) Find the largest integer n that satisfies $\frac{3}{\left|3-\frac{n}{7}\right|} \ge 4$.

A) 23 B) 25 C) 26 D) 28 E) NOTA

7) How many distinct positive values of y satisfy $27^y - 9^{y+\frac{1}{2}} + 3^{y+1} = 1$?

A) 0 B) 1 C) 2 D) 3 E) NOTA

8) For how many positive integers k does there not exist a pair of nonnegative integers (m,n) such that $3m + 7n = k$?

A) 5 B) 6 C) 7 D) 8 E) NOTA

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9) Solve for x: $\frac{x}{x-2}+\frac{1}{x-1}\geq 1$.

A) $x<1, 4/3 \leq x<2$ B) $x\leq 1, 4/3 \leq x \leq 2$ C) $x\geq 2, 4/3 \geq x \geq 1$

D) $x>2, 4/3 \geq x>1$ E) NOTA

10) How many integers x satisfy both $3x^2 \leq 307$ and $4x^3 \leq 37$?

A) 4 B) 5 C) 12 D) 13 E) NOTA

11) What is the area contained in the intersection of the graphs of: $|x|+|y|\geq 2$ and $x^2+y^2\leq 4$?

A) $4\pi - 8$ B) $16 - 4\pi$ C) $4\pi - 4$ D) $16\pi - 16$ E) NOTA

12) Let r, s, and t be the solutions to the equation $x^3-16x^2+67x-70=0$, where $r>s>t$. Find r-s-t.

A) 2 B) 2 1/3 C) 3 D) 4 E) NOTA

13) Let r, s, t, u be the solutions of

$$(x-1)(x-2)(x-3)(x-4)=0$$

and let a, b, c, d be the solutions of

$$(y-5)(y-6)(y-7)(y-8)=0.$$

Find the maximum possible value of $au + bt + cs + dr$.

A) 60 B) 64 C) 70 D) 84 E) NOTA

14) If r, s, t are the solutions to the equation $4x^3-5x^2-6x+8=0$, find $r^2s^2+s^2t^2+r^2t^2$.

A) 11/4 B) 4 C) 29/4 D) 8 E) NOTA

15) How many sets of integer solutions are there to:

$$3x^2-2y^2+z^2=37$$
$$-2x^2+3y^2-4z^2=-108?$$
$$-4x^2-y^2+z^2=9$$

A) 2 B) 4 C) 6 D) 8 E) NOTA

16) Find the sum of the ninth powers of the six sixth roots of -2.

A) -2 B) 0 C) $-12\sqrt{2}$ D) $12\sqrt{2}$ E) NOTA

17) What is the smallest possible value of $x^4+4x^3+7x^2+6x+9$ for real values of x?

A) 3 B) 5 C) 7 D) 9 E) NOTA

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18) For how many real values of y is $10^y - 9^y - 8^y = 137$?

A) 0 B) 1 C) 2 D) infinitely many E) NOTA

19) What is the smallest possible value of $32n^2-127n+14$ for integer values of n?

A) -114 B) -81 C) -144 D) 14 E) NOTA

20) How many quadratic expressions of the form ax^2+bx+c have integer roots, integer coefficients, and $a=1$, $5>b,c>-5$?

A) 19 B) 21 C) 25 D) 27 E) NOTA

21) Find $f(2)$ if $f(1/x) - x^2 = (1-x)f(x)$.

A) -3/4 B) -3/2 C) 3/2 D) 3/4 E) NOTA

22) Given that $a_1, a_2, a_3, ..., a_{10}$ are positive integers such that $\sum_{i=1}^{10} \frac{1}{a_i} \geq 2.3$, what is the largest possible value of $\min(\{a_i\})$, which is defined to be the smallest member of the set $\{a_1, a_2, a_3, ..., a_{10}\}$?

A) 3 B) 4 C) 5 D) 6 E) NOTA

23) Given that $f(x) = \frac{x}{x+1}$, then solve for x: $f(f(f(f(f(f(x)))))) = 1$.

A) 1/5 B) -1/5 C) 1 D) no solution E) NOTA

24) It takes 8 people 5 days to build 3 huts. How many days does it take 7 people to build one hut if all seven start on work on the first day and one person quits at the end of each day?

A) 2 1/3 B) 2 1/15 C) 1 19/21 D) 2 1/21 E) NOTA

25) When factored into as many polynomials as possible of degree one or greater with real coefficients, how many factors does x^7+1 have?

A) 2 B) 4 C) 6 D) 7 E) NOTA

26) If $x^n + a = 0$ has multiple distinct real solutions x for positive integer n and real number a, what must be true about n and/or a?

I) $a > 0$
II) $a < 0$
III) n is even
IV) n is odd

A) II and III together B) I and IV together C) I and III together D) II only E) NOTA

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27) Given that $\sum_{i=1}^{6} ia_i = 36$ and all a_i are positive, what is the largest possible value of $\prod_{i=1}^{6} a_i$?

A) 64.8 B) 66.4 C) 68.6 D) 72.2 E) NOTA

28) If $y = \sqrt{2x^2 + 5x - 3}$ and $y = k(2x-1)$, then one solution is $(x, y) = (1/2, 0)$ for any k. Find the value of x in terms of k for $y \neq 0$.

A) $x = k^2 - 3$ B) $x = \dfrac{7k}{2k^2 - 1}$ C) $x = \dfrac{k^2}{2k^2 - 1}$ D) $x = \dfrac{k^2 + 3}{2k^2 - 1}$ E) NOTA

29) Find $x + y + z$ if

$$x\sqrt{y} = 216\sqrt{3}$$
$$y\sqrt{z} = 240\sqrt{6}$$
$$z\sqrt{x} = 450\sqrt{6}$$

A) 248 B) 252 C) 256 D) 262 E) NOTA

30) Find the sum of all real x which satisfy $x^6 + x^4 - 115x^3 + x^2 + 1 = 0$.

A) 0 B) 115 C) 3 D) 5 E) NOTA