Precalculus Team Questions

January Regional 2003

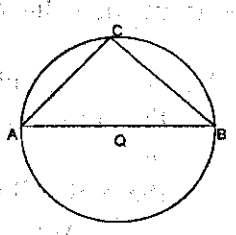
1. Let $f(x) = 8\sin(3x-2)+4$ for all x>0 (x is measured in radians).
Let 2A = the period of f(x).
Let B = the minimum value of f(x).
Let C = the smallest value of x such that f(x)=0.
Determine the value of $\frac{C}{A+B}$ in simplest terms.

2. Acute triangle ABC has area 27 square units. If $AB = 11$ and $AC = 8$ then what is the measure of angle C to the nearest degree?

3. A parabolic arch is 9ft high and 8ft across at the base. A bird flies Xft (in a straight line) from a point on the arch 5ft above ground level to a point on the arch 8ft above ground level. What is the largest possible value of X?

4. Let $f(x) = x^{x+1}$. Evaluate the following expression:
$f(-4) \bullet f(-3) \bullet f(-2) \bullet f(-1) \bullet f(1) \bullet f(2) \bullet f(3)$

5. $\overline{AB}$ is a diameter of circle Q in the picture (which is not drawn to scale). If the measure of angle A is equal to the measure of angle B and the area of triangle ABC is 22 square units, then give the area of Q in square units. (Note: C is a point on circle Q.)



6. Let $f(x) = (A+1)x^2 - (A+3)x + (A+4)$ for some constant A.
If $f(A) = A^3$ then determine the value of A.

7. Forty-five people were interviewed about their snacking habits. Fifteen have a mid-morning snack, sixteen have a mid-afternoon snack, three have a mid-morning and mid-afternoon snack, seven have a mid-morning and midnight snack, five have a mid-afternoon and midnight snack, two have all three snacks and five don't snack at all. How many of the interviewed people have a midnight snack?

8. Let PQP_7, written in base 7, equal $P07_8$, written in base 8.
(Note: 0 in 'P07' denotes 'zero', not the letter O.)
Let PQP_6, written in base 6, equal $1PQ_9$, written in base 9.
If P and Q are each whole numbers less than 10, determine P+Q.

Precalculus Team Questions continued

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9. A square of area 144 square units is drawn. A circle is inscribed in the square and a second square is inscribed in the circle. A point is tossed and lands within the outer square. Let P equal the probability that the point lands inside the circle but outside the inner square. Determine the value of 4P.

10. On January 1, 2001, I deposit $10 into a bank account paying 5% annual compounded interest (paid annually on December 31). On January 1, 2002, I deposit $20 into the account. If, on each January 1 of successive years, I deposit an amount $10 more than I deposited the prior year, what will my account balance (including interest) be after the annual interest is paid on December 31, 2005? Answer to the nearest penny.

11. Let $P(x,y) = \log_x y$ and $Q(x) = \frac{x-1}{x-4}$.

Let A = Q(P(3,27)).
Let B = P(Q(5),4)).
Let Q^{-1}(C)=P(1-A,B).
Determine the value of C.

12. Let A = the number of zeros in which 2003! ends.
Let B = the number of distinct permutations of the letters HYPERBOLA taken 3 at a time.
Let C = the probability of randomly selecting a T from the following twelve letters: M, U, A, L, P, H, A, T, H, E, T, and A.
A-BC = ?

13. Let f(x) be defined as the difference when 5 is subtracted from 3x. Determine the summation from x=1 to x=10 of f(x).

14. Let $\frac{3}{x-3}+\frac{-2}{x+2}=\frac{Ax+B}{(x+2)(x-3)}$. If $f(x,y) = x^y$ then determine the value of f(A,B).

15. A parabola with vertex at the origin has directrix $x = -4$. Find the sum of all values of Z such that the point (5,Z) is on the graph of the parabola.